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ROOT PLACEMENT WITH
TRANSFER FUNCTION METHODS

by

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Thesis Advisor: George J. Thaler

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Root Placement with Transfer Function Methods

by

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Major, United States Army
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Submitted in partial fulfillment of the requirements for the degree of

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September 1988

ABSTRACT

A general design technique for root placement with full state feedback using transfer function methods is presented. The design procedure presented is applicable for linear, time invariant (LTI) systems in the s-domain for continuous time. This general design technique is then used to develop and explore design procedures for root placement with partial state feedback. Numerous system examples are presented to demonstrate the procedure with this design technique when less than all the system states are available to be measured and fedback. Both the all pole plant and the plant with a zero are considered.

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I. INTRODUCTION

Many physical systems may be modeled by differential equations as single-input, single-output systems. The differential equations, with initial conditions, can be transformed to the s-domain to yield algebraic equations with the complex variable s. These algebraic equations in the s-domain may be written as a ratio of output to input to yield a system transfer function. System plants are often described by transfer functions in the s-domain. To actually design, build and test a plant for a specific purpose can be a very time-lengthy endeavor. Usually a plant is "bought off the shelf" to perform a particular task, and a specific control system is designed to drive and/or restrict that system as applicable. In many cases the "control design" of the system follows the "application design" of the system. The concept of feedback plays a major role in most automatic control systems.

The feedback design procedures presented in this research paper are applicable to linear, time invariant (LTI) systems in continuous time. It is assumed that the reader is familiar with classical control design tools, i.e., root locus, BODE, and NYQUIST. The time performance of a system is a critical design factor in an automatic control system.

The time performance criteria of a system is usually measured as a function of the initial overshoot and number of oscillations to a given input, and the settling time of the transient response. Dominant roots are chosen to satisfy the time specifications of a system. Figure 1.1 and Appendix A show the key parameters for controlling a second order system.

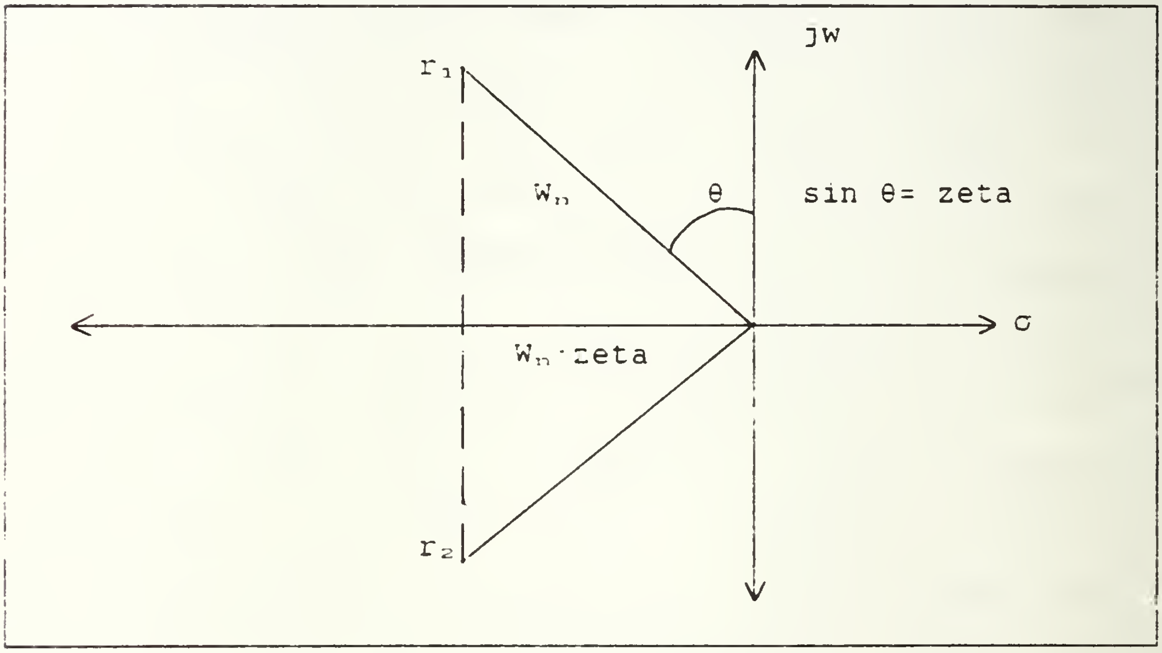

Figure 1.1 Dominant Roots

The number of feedback states in a system is usually a function of the following parameters:

1. cost
2. weight
3. size
4. application

Chapter II presents a design technique for root placement with full state feedback using transfer function methods for the all pole plant when only N-1 roots may be chosen by the designer. The final results using this design procedure are equivalent to those using classical state variable analysis techniques when all N roots are chosen by the designer. Chapter III further develops the design procedure for root placement with partial state feedback using transfer function methods for the all pole plant. With this design procedure less than all the states are available to be measured and fedback. The design procedures developed in Chapter III for partial state feedback using transfer function methods does not use an observer. An observer is used to control a system when less than all of the states are available to be measured and fedback. An observer may be defined generally as a physical feedback device that uses measured data from the input, output, and accessible states of a system, to estimate those states that are not directly accessible to be measured and fedback. Chapter IV extends the design procedure for the plant with a zero.

The procedures developed in Chapters II-IV are effective design procedures in many cases. However, they will not always satisfy the given system specifications. In such cases, a combination of the design procedures presented in this paper and other compensation schemes should be considered by the engineer.

II. FULL STATE FEEDBACK: ALL POLE PLANT

A. INTRODUCTION

Designing a control system using root placement with full state feedback requires that all of the states are available to be measured and fedback. With classical state variable root placement methods, all roots of the system must be specifically located by the designer. Root placement with full state feedback utilizing transfer function methods, however, requires that only N-1 roots be specifically located by the system designer. The unspecified root will follow an asymptotic angle of -180° towards infinity as the gain of the system approaches infinity. The number of excess poles in a system is not changed by state feedback. With full state feedback using transfer function methods, the system output and the output's N-1 derivatives are the feedback states [Ref. 1:p. 1]. The general concepts developed for full state feedback in this chapter are applicable to partial state feedback design techniques considered in subsequent chapters.

B. GENERAL CONCEPT

As is generally the case with any system that is to be compensated or modified, the first step is to completely evaluate the uncompensated system. This initial system evaluation would include, but should not be limited to, finding the system roots and error coefficient, and obtaining

the uncompensated BODE diagram and root locus plot. It is assumed that it is known <u>how</u> the compensated system is to perform, i.e., system specifications have been given and a pair of dominant roots have been selected to ensure system stability, accuracy and desired transient response behavior. The required time performance specifications of the system usually determine the location of the dominant roots. The dominant roots of a system are defined in a general sense as those roots of the closed loop system with the smallest real value.

Using root placement with transfer function methods, the system output and the output's N-1 derivatives are the feedback states. Figure 2.1 shows the basic block diagram for state feedback.

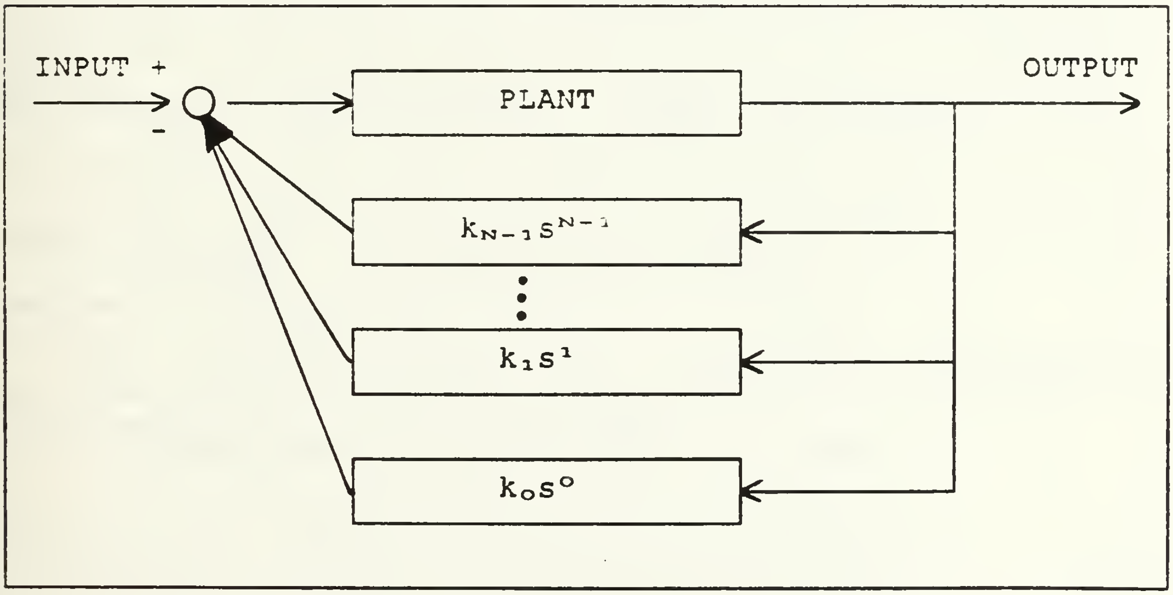

Figure 2.1 Basic State Feedback Block Diagram

The feedback states shown in Figure 2.1 may be combined into a feedback polynomial as shown in Figure 2.2.

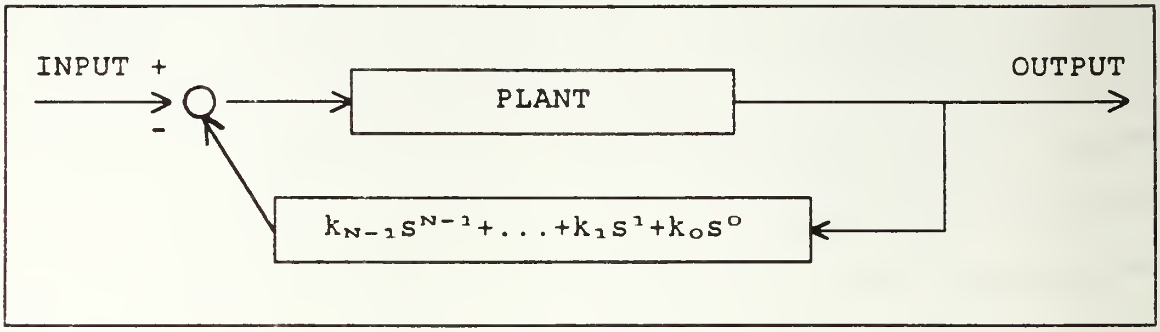

Figure 2.2 Reduced State Feedback Block Diagram

Figure 2.3 shows that the reduced state feedback block diagram in Figure 2.2 may be further manipulated to preserve unity feedback by placing the coefficient of the zeroth derivative in the forward path.

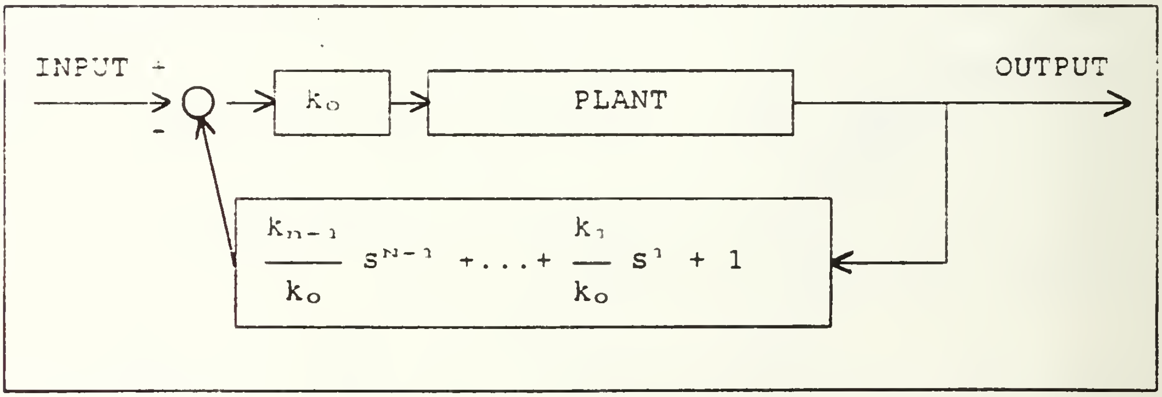

Figure 2.3 Unity Feedback Preservation

The plant can be defined in general terms as

$$G(s) = \frac{K}{s^N + C_{N-1}s^{N-1} + C_{N-2}s^{N-2} + \ldots + C_1 s^1 + C_0} \tag{2.1}$$

and from Figure 2.2, the feedback polynomial is

$$H(s) = k_{N-1}s^{N-1} + \ldots + k_1 s^1 + k_0 \tag{2.2}$$

Thus the characteristic equation for the compensated system becomes

$$s^N + (C_{N-1} + Kk_{N-1})s^{N-1} + \ldots + (C_1 + Kk_1)s + (C_0 + Kk_0) = 0 \tag{2.3}$$

From the theory of equations it is known that the roots of any polynomial are functions of the polynomial's coefficients and conversely the polynomial's coefficients are a function of that polynomial's roots. From inspection of equation 2.3 it is clear that every root of the system may be specifically located by adjustment of the feedback coefficients. Therefore if N-1 roots of the characteristic equation are chosen, the coefficients of this polynomial are the desired feedback gains [Ref. 1:p. 1-2].

The G(s)H(s) function can be written in factored form from equations 2.1 and 2.2 to yield

$$G(s)H(s) = \frac{K(s + r_{H(s)1})(s + r_{H(s)2}) \ldots (s + r_{H(s)N-1})}{(s + r_{G(s)1})(s + r_{G(s)2}) \ldots (s + r_{G(s)N})} \tag{2.4}$$

Equation 2.4 shows that the system has one excess pole and the zeros of the H(s) function define the system designer's desired root locations! The root loci of the system will start at the open loop pole locations and end on the zeros as the gain of the system approaches infinity. The unspecified root must be real and moves on the negative real axis towards infinity [Ref. 1:p. 6]. The loop gain required to move the roots to the zeros of the G(s)H(s) function will approach infinity as the roots approach the zeros. This extremely high loop gain is usually not realizable and drastically changes the error coefficient of the system. However, by offsetting the zeros that are attracting the chosen dominant roots, the system loop gain can be significantly reduced. The zero offset locations are chosen by extending the path of the root loci past the desired dominant root locations. Only the zeros that are attracting the dominant roots are offset. The zero offset procedure is a trial and error procedure that is dependent on the particular system under design.

EXAMPLE 2.1 Equation 2.5 defines the plant for a fourth order system.

$$G(s) = \frac{K}{s(s+5)(s+5)(s+10)} \tag{2.5}$$

Dominant root locations have been chosen at s= $-2\pm j2$ to satisfy required system time performance and bandwidth specifications. The uncompensated root locus plot is shown

in Figure 2.4.a. With all of the states available to be fedback, the designer may name N-1 roots. The third specifiable root will be placed at s= -9 to maintain a dominant role for the two dominant roots at s= -2±j2. The G(s)H(s) function then becomes

$$G(s)H(s) = \frac{K(s+2+j2)(s+2-j2)(s+9)}{s(s+5)(s+5)(s+10)} \tag{2.6}$$

and the compensated root locus plot is shown in Figure 2.4.b. An extremely high gain is required to move the roots to the zeros of the G(s)H(s) function. Table 2.1 shows the gain required to move the specified roots to their desired locations and the location of the fourth unspecified root.

TABLE 2.1 LOOP GAIN AND ROOT LOCATIONS

K	r_1	r_2	r_3	r_4
1.0	-0.285	-3.728	-6.658	-10.329
10.0	-1.842±j1.170		-8.704	-17.612
50.0	-1.931±j1.842		-8.963	-57.175
75.0	-1.948±j1.905		-8.982	-82.122
100.0	-1.958±j1.936		-8.992	-107.093
1,000.0	-1.987±j2.023		-9.016	-1007.010
10,000.0	-1.990±j2.023		-9.019	-10007.001

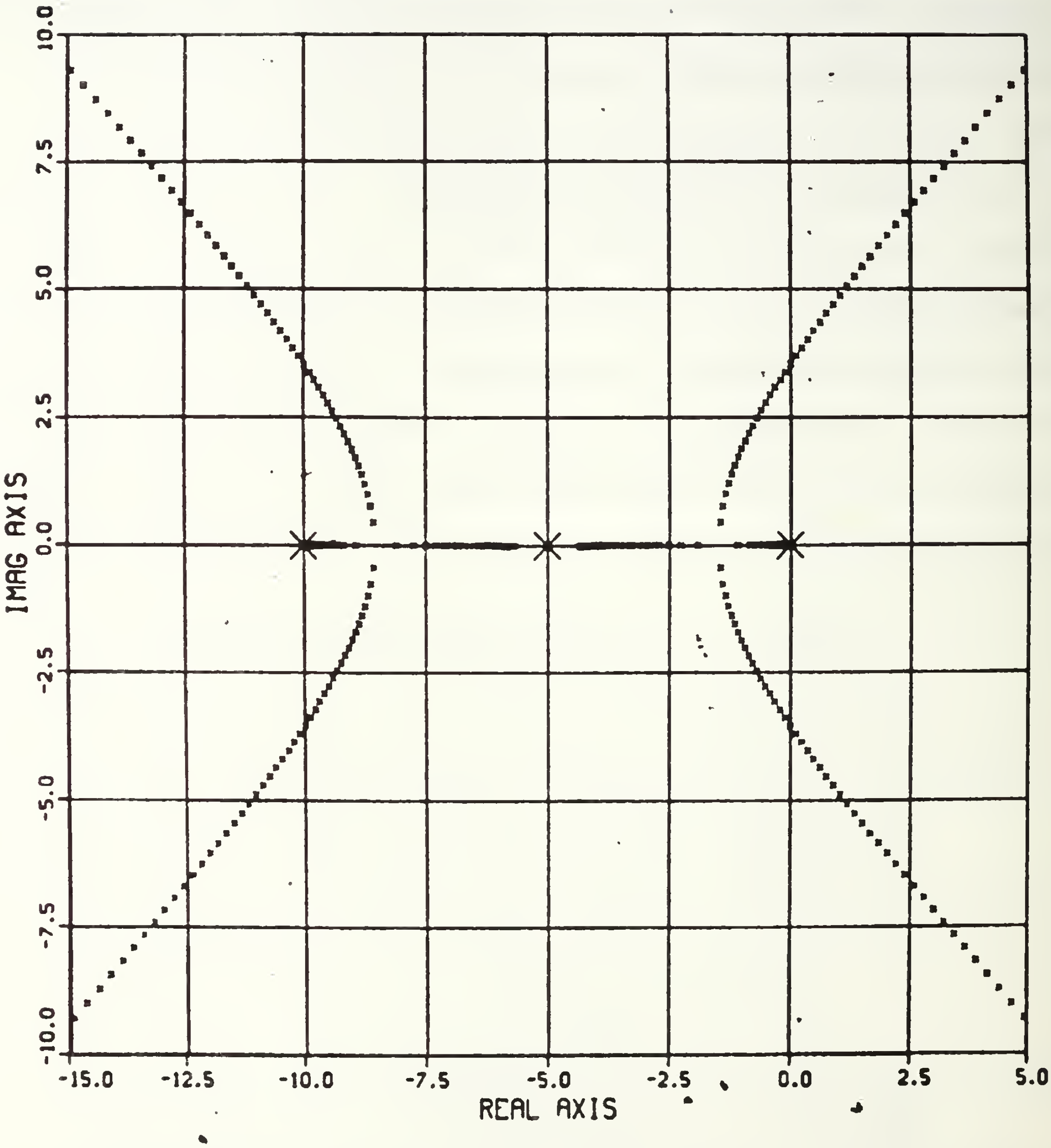

Figure 2.4.a Uncompensated System Root Locus Plot

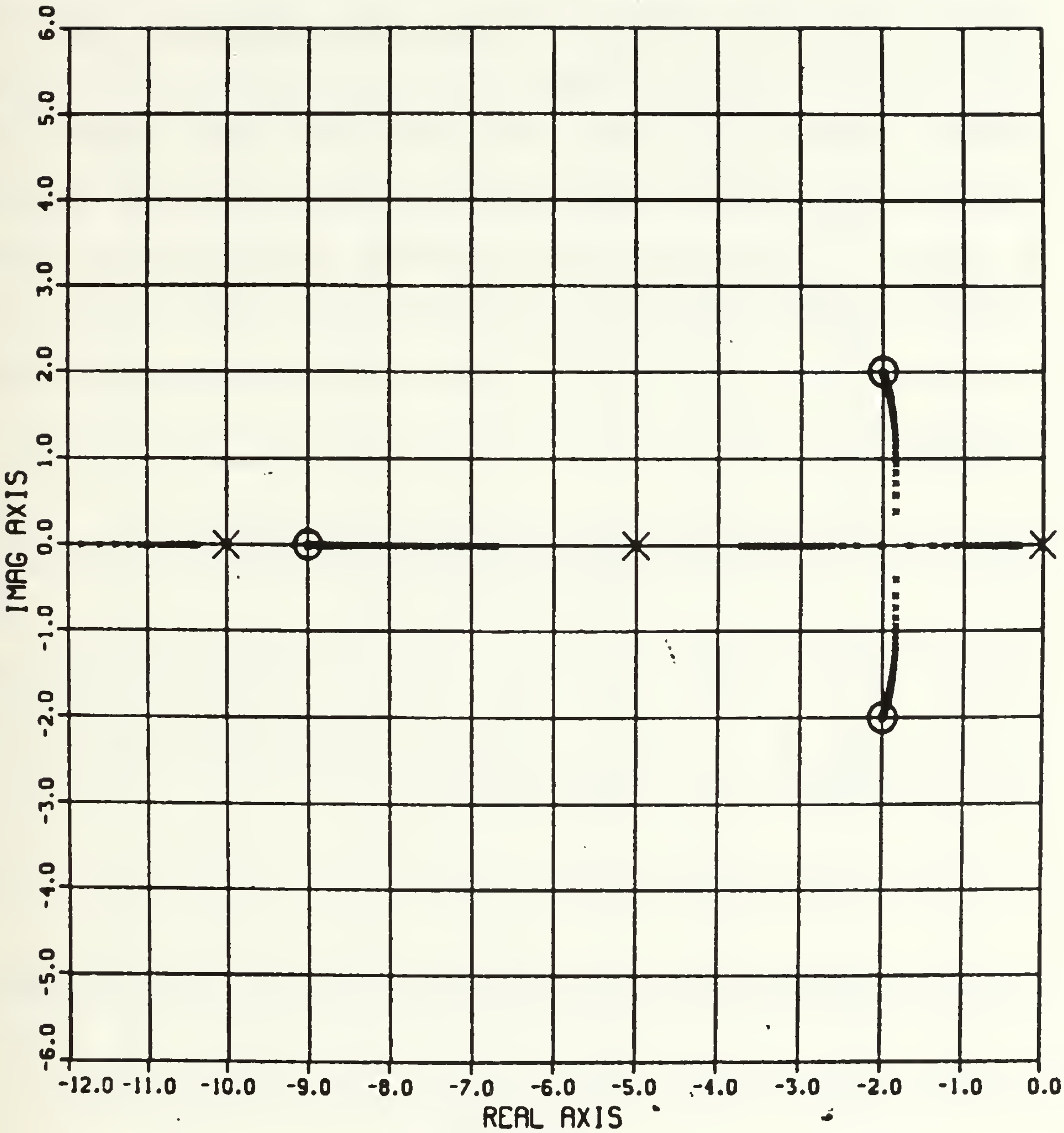

Figure 2.4.b Compensated System Root Loci without Zero Offsets

Clearly the unspecified root (r_4) moves along the negative real axis towards infinity as the loop gain is increased. To maintain the system error coefficient and have a realizable system gain, zero offset locations are chosen for the zeros that attract the dominant roots. Figure 2.4.b shows the general shape of the root loci as the roots approach the zeros of the G(s)H(s) function. By trial and error and using a computer aided design (CAD) program, acceptable zero offset locations are found at s= -2.5±j3.0 such that the root loci pass through s= -2.0±j2.0. The modified G(s)H(s) function becomes

$$G(s)H(s) = \frac{k(s+2.5+j3.0)(s+2.5-j3.0)(s+9)}{s(s+5)(s+5)(s+10)} \qquad (2.7)$$

Figure 2.4.c shows the root locus plot with zero offset locations at s= -2.5±j3.0. Using a loop gain equal to 7.3, the closed loop roots are located at

$$s= -2.003\pm j1.99,\ -8.53,\ -14.8 \qquad (2.8)$$

and the compensated block diagram is shown in Figure 2.4.d.

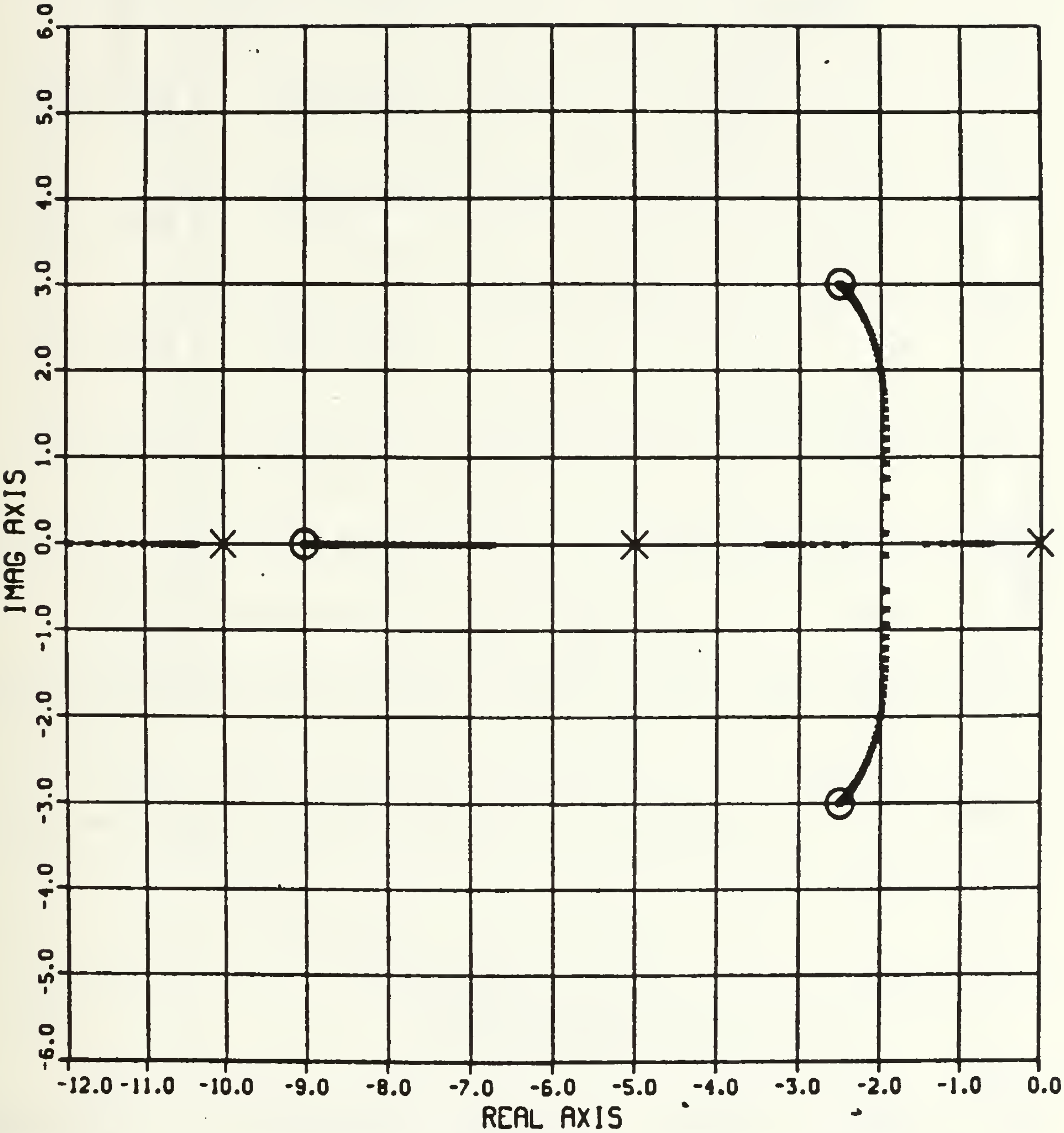

Figure 2.4.c Compensated System Root Loci with Zero Offsets

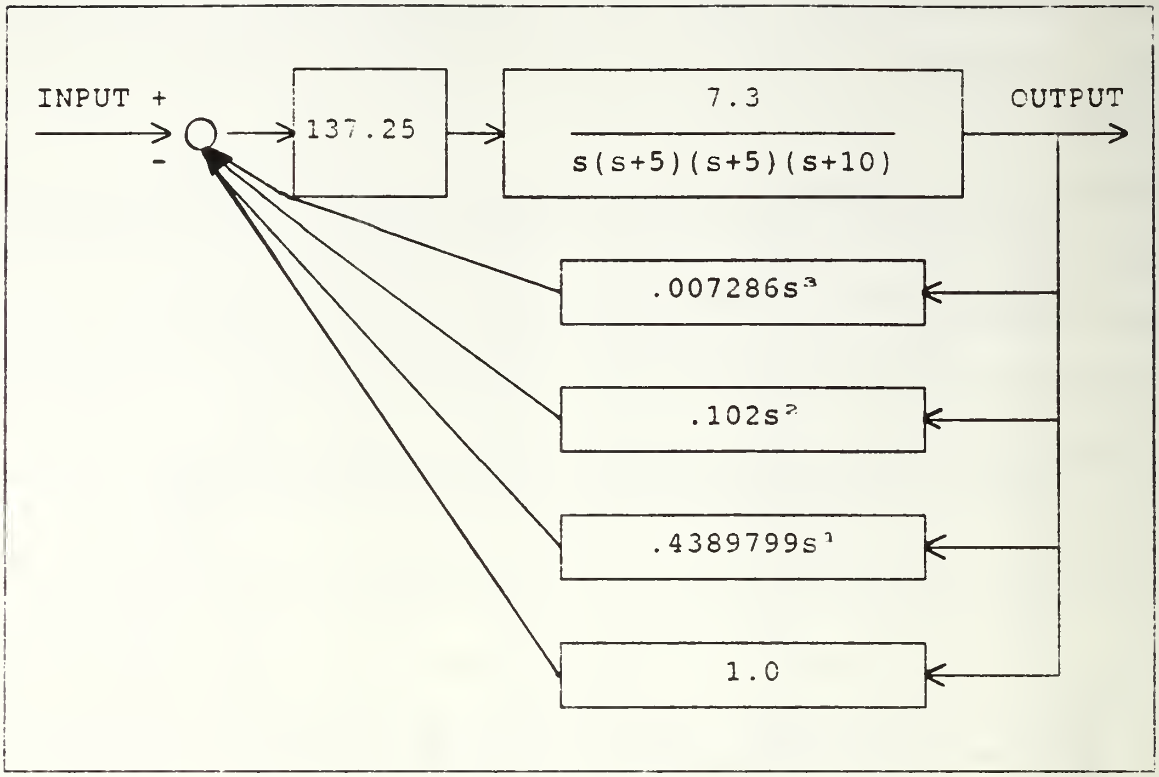

Figure 2.4.d Compensated System Block Diagram

The compensated system step response and BODE diagram are shown in Figures 2.4.e and 2.4.f respectively.

EXAMPLE 2.2 The plant for a sixth order system is defined by equation 2.9.

$$G(s)= \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)} \tag{2.9}$$

Once again the dominant roots are required to be located at s= -2±j2 to meet the given system specifications. The

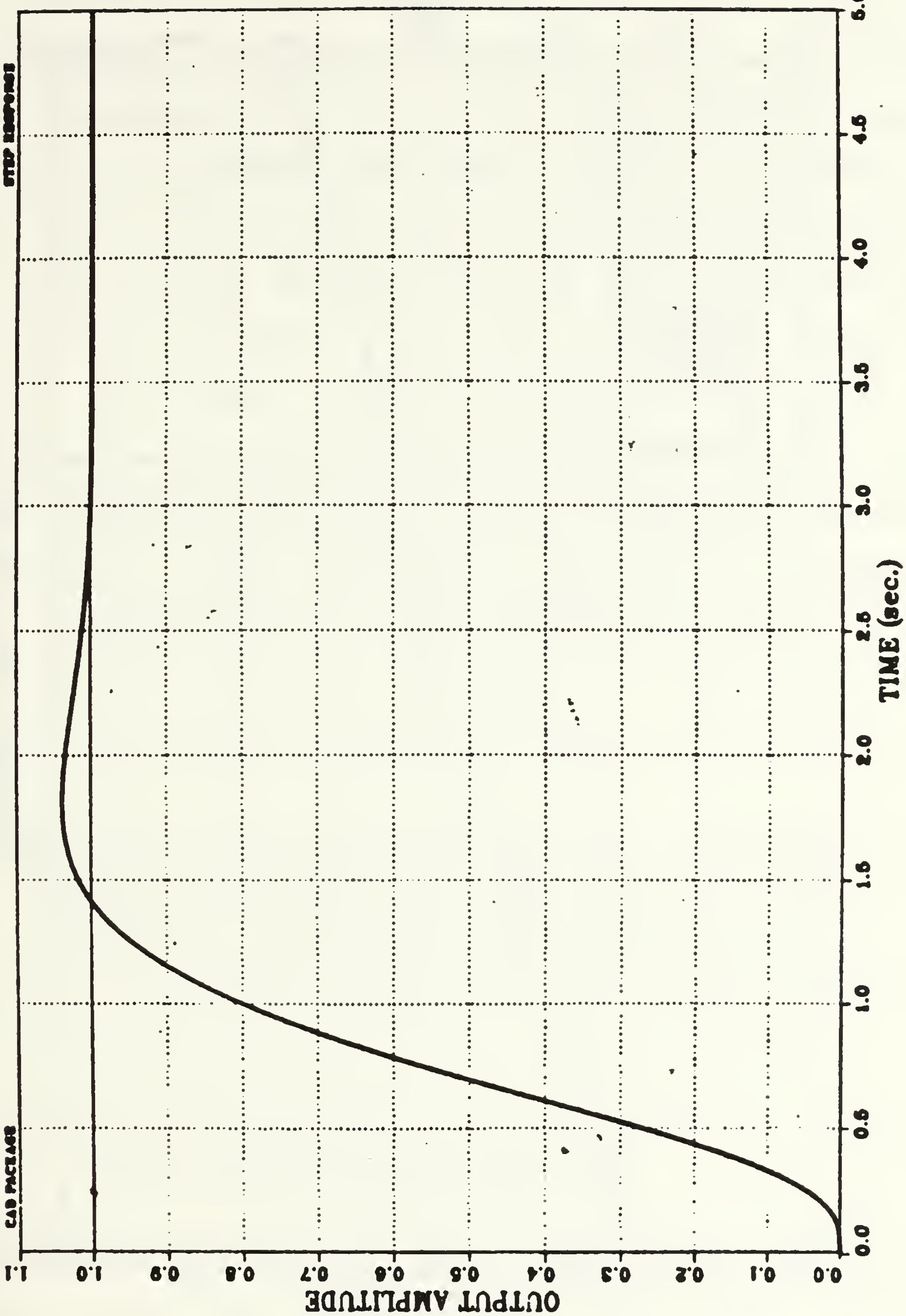

Figure 2.4.e Compensated System Step Response

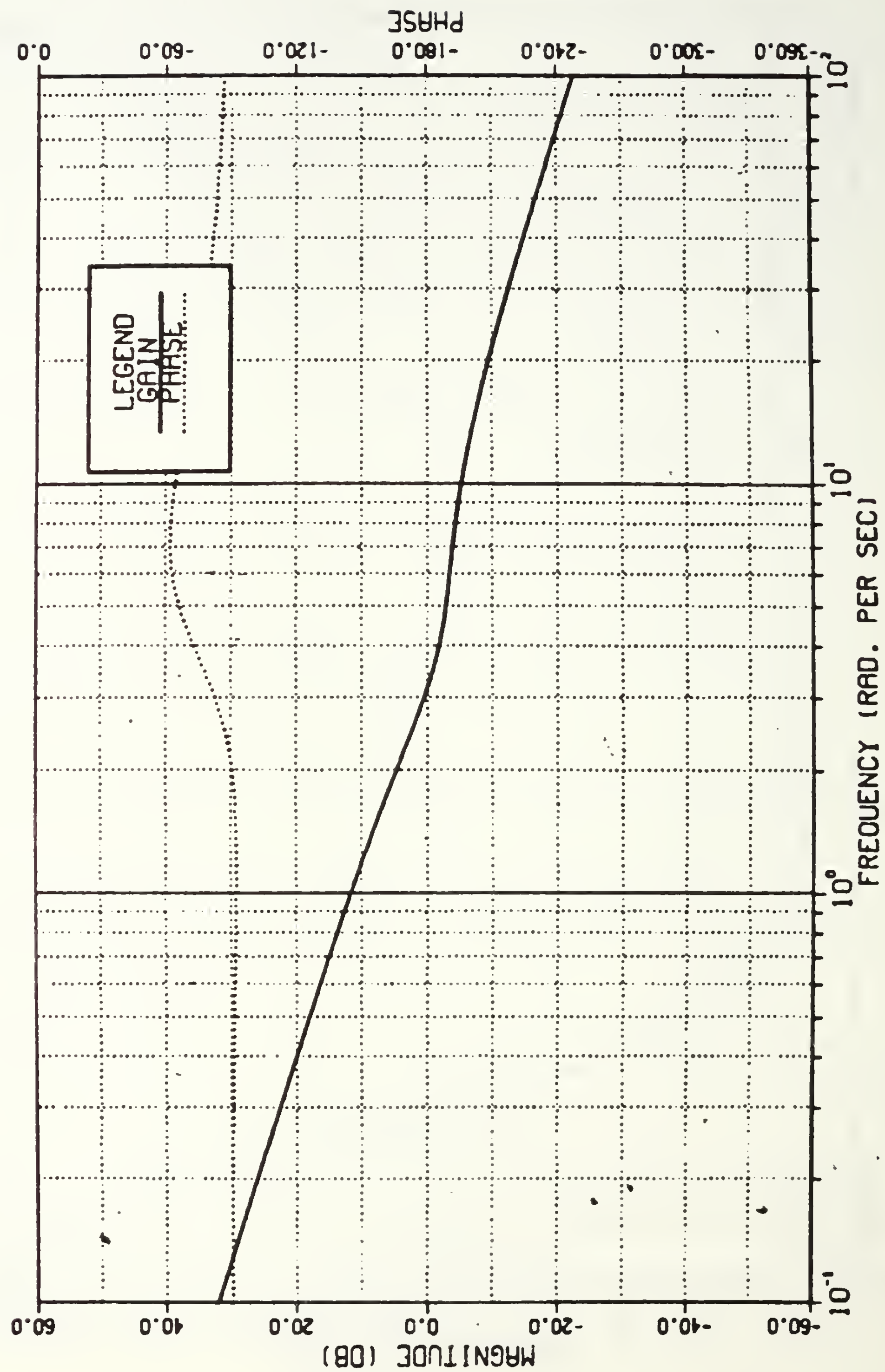

Figure 2.4.f Compensated System BODE Diagram

remaining specifiable root locations are chosen such that the roots move as little as possible from their natural open loop pole locations, yet allow the dominant roots to retain their dominant system role [Ref. 2]. As the compensated system loop gain is increased the roots follow the loci shown in Figure 2.5.a. By trial and error, zero offset locations are found at s= -5.0±j.8 such that the root loci pass through the dominant root locations at s= -2±j2. The root locus plot with the zero offset locations is shown in Figure 2.5.b for the dominant roots. With a root locus gain of 26.5, the system's dominant roots are located at s= -2±j2 as depicted in Figure 2.5.c. The compensated system block diagram is shown in Figure 2.5.d. The compensated system roots are located at

$$s= -1.97\pm j2.05,\ -4.87,\ -5.12,\ -9.85,\ -73.7 \qquad (2.10)$$

The compensated system step response and BODE diagram are shown in Figures 2.5.e and 2.5.f respectively.

EXAMPLE 2.3 A seventh order plant is defined by equation 2.11.

$$G(s)= \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \qquad (2.11)$$

The uncompensated root locus plot is shown in Figure 2.6.a. Dominant roots are chosen at s= -2±j2 to meet the time performance specifications of the system. The remaining

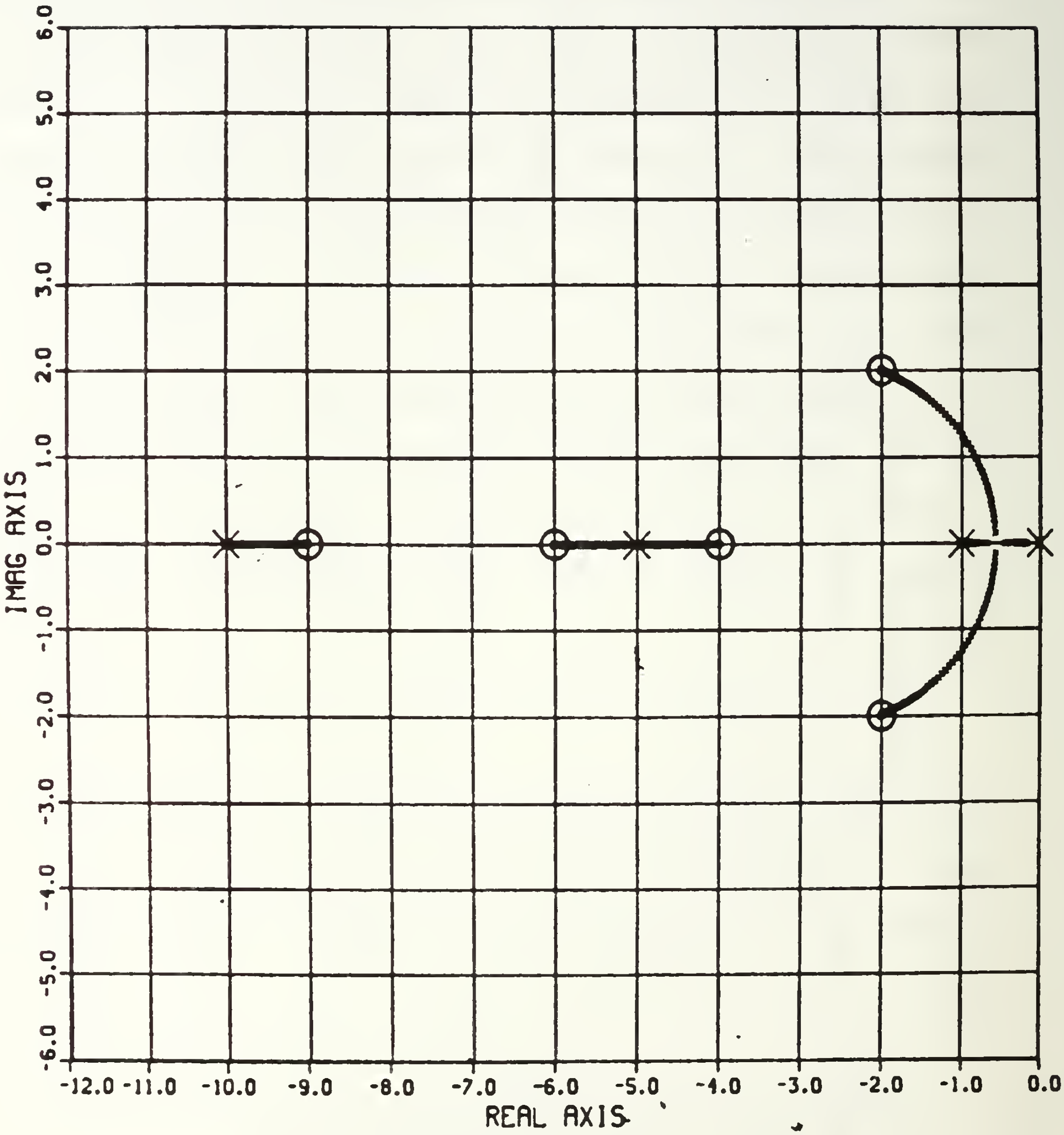

Figure 2.5.a Compensated System Root Loci without Zero Offsets

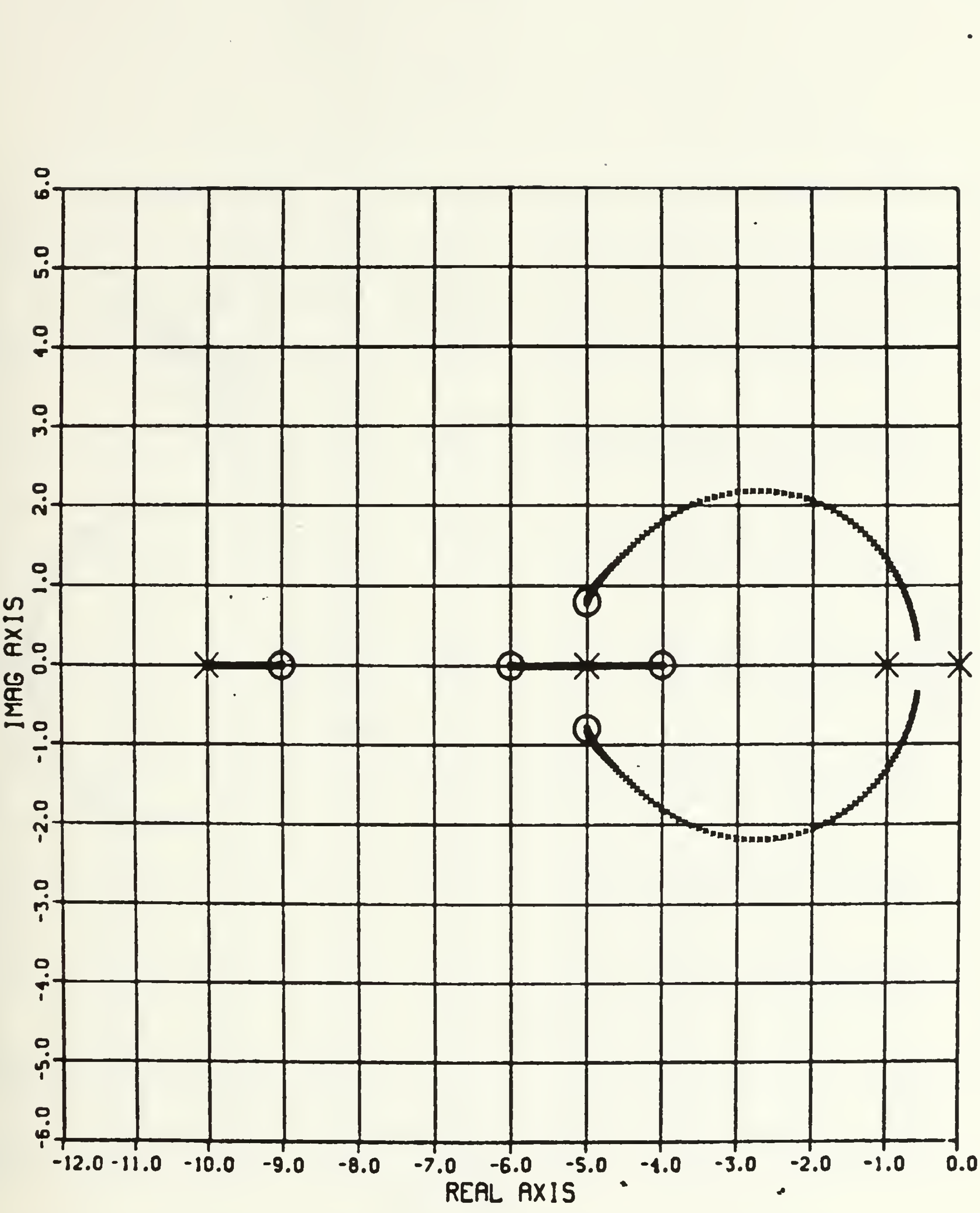

Figure 2.5.b Compensated System Root Loci with Zero Offsets

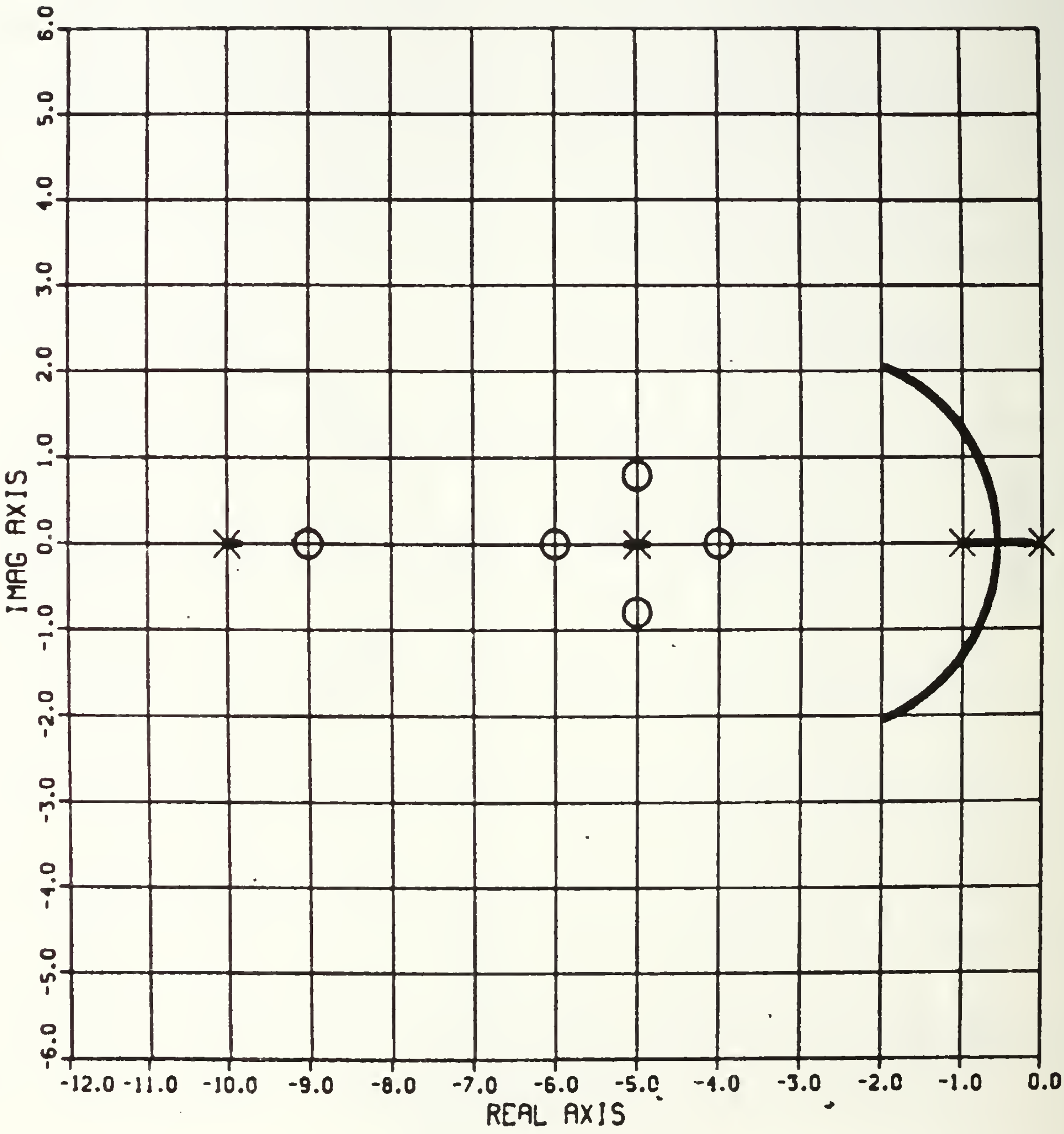

Figure 2.5.c Final Compensated System Root Locus Plot

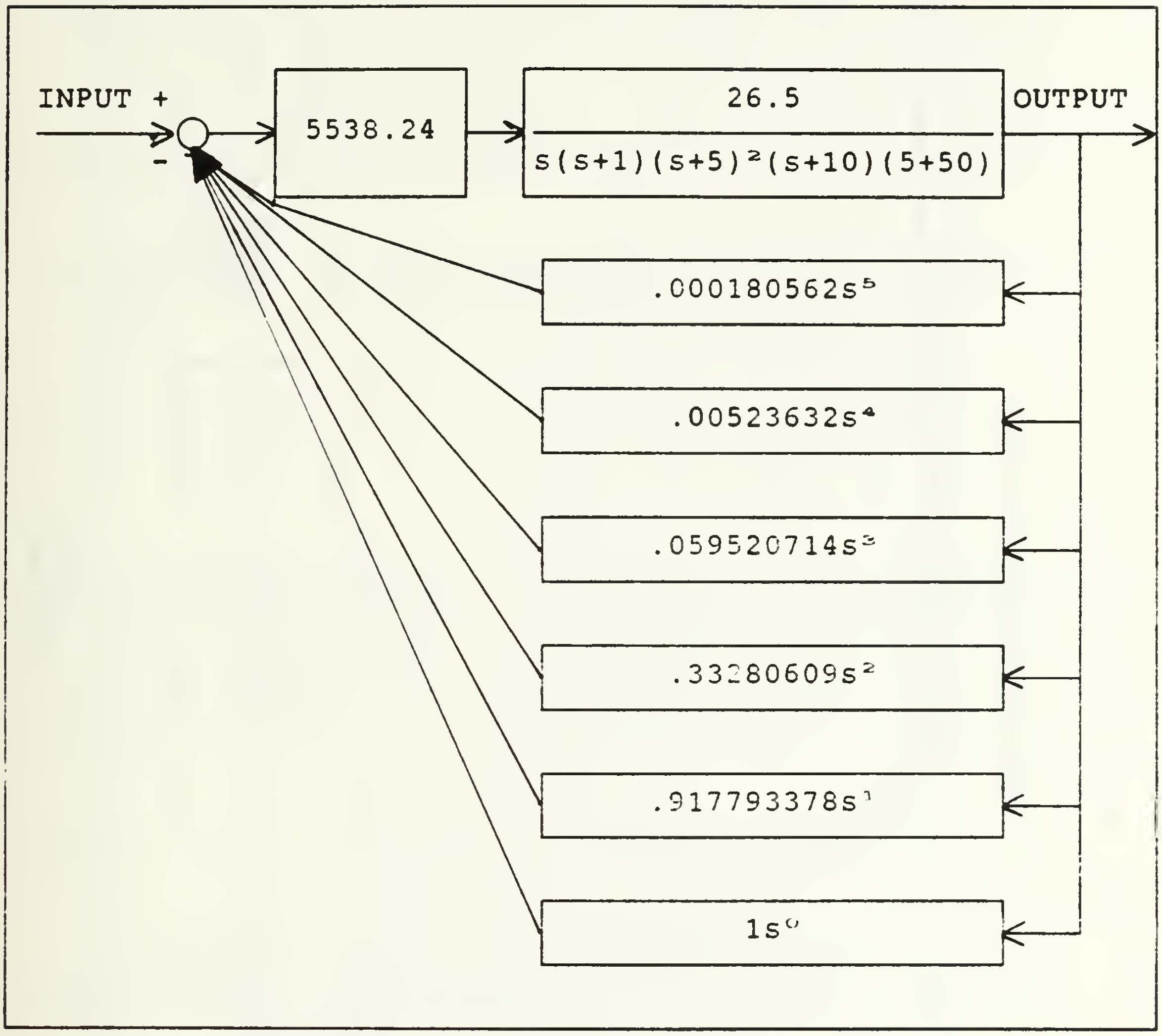

Figure 2.5.d Compensated System Block Diagram

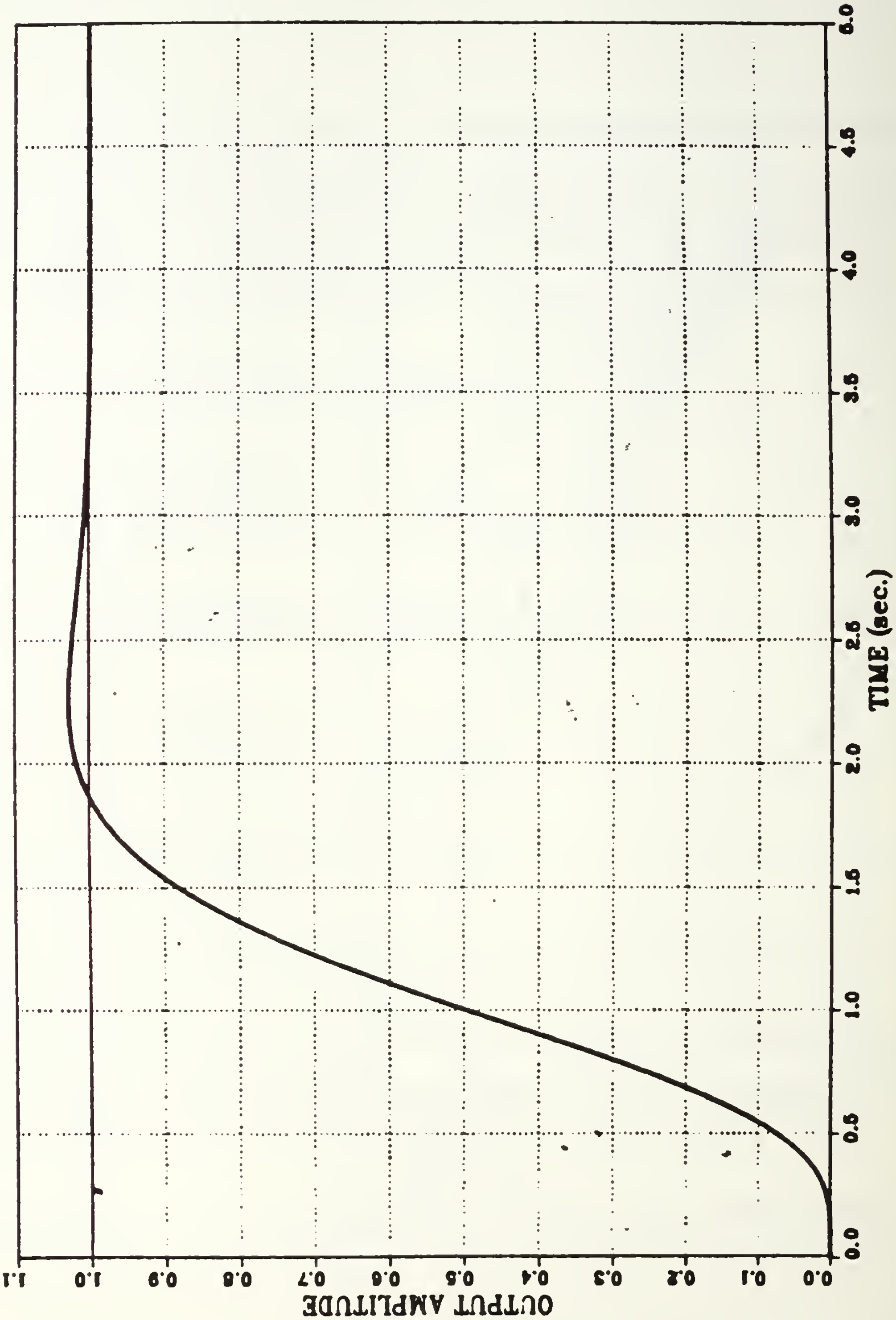

Figure 2.5.e Compensated System Step Response

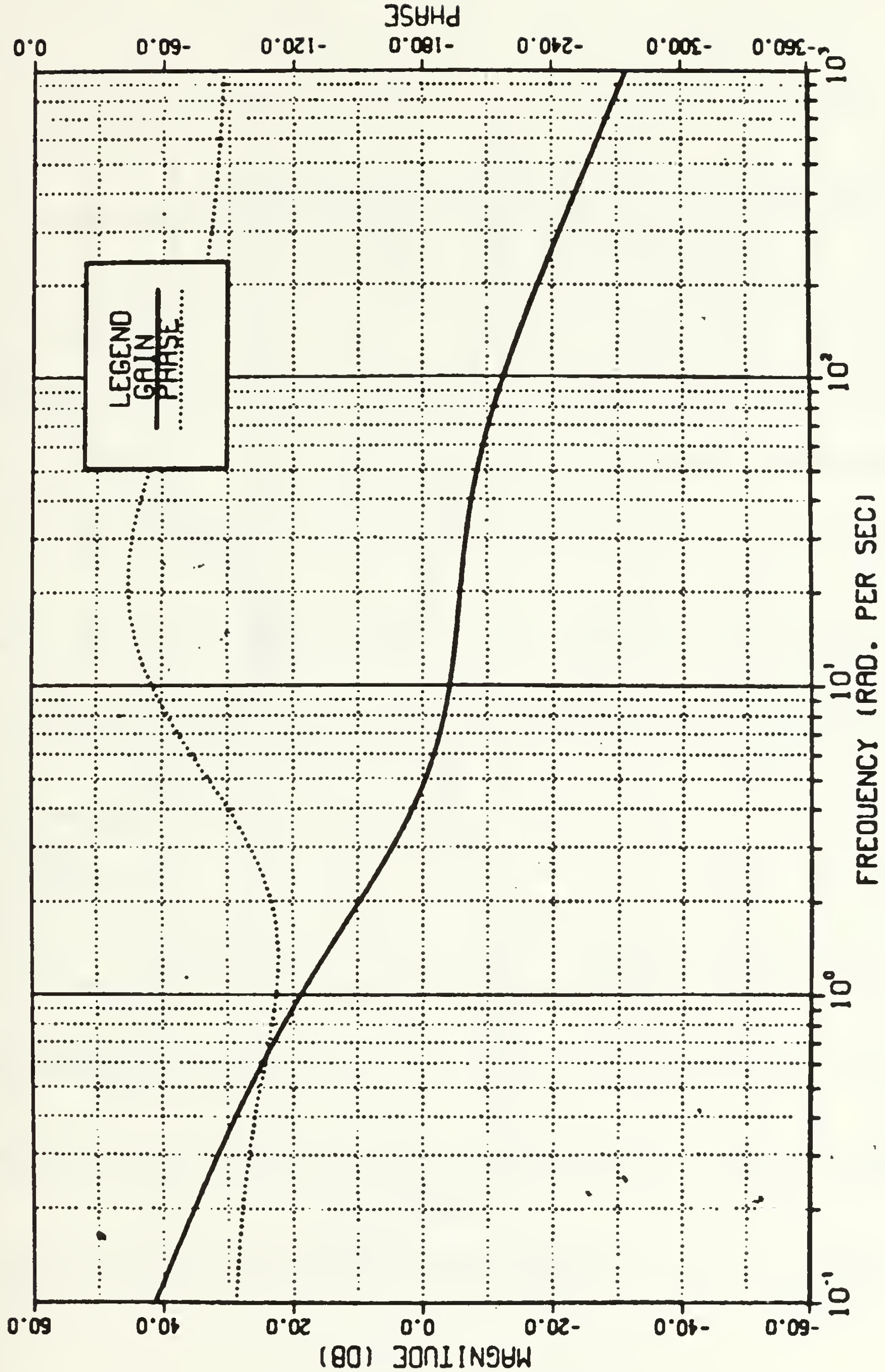

Figure 2.5.f Compensated System BODE Diagram

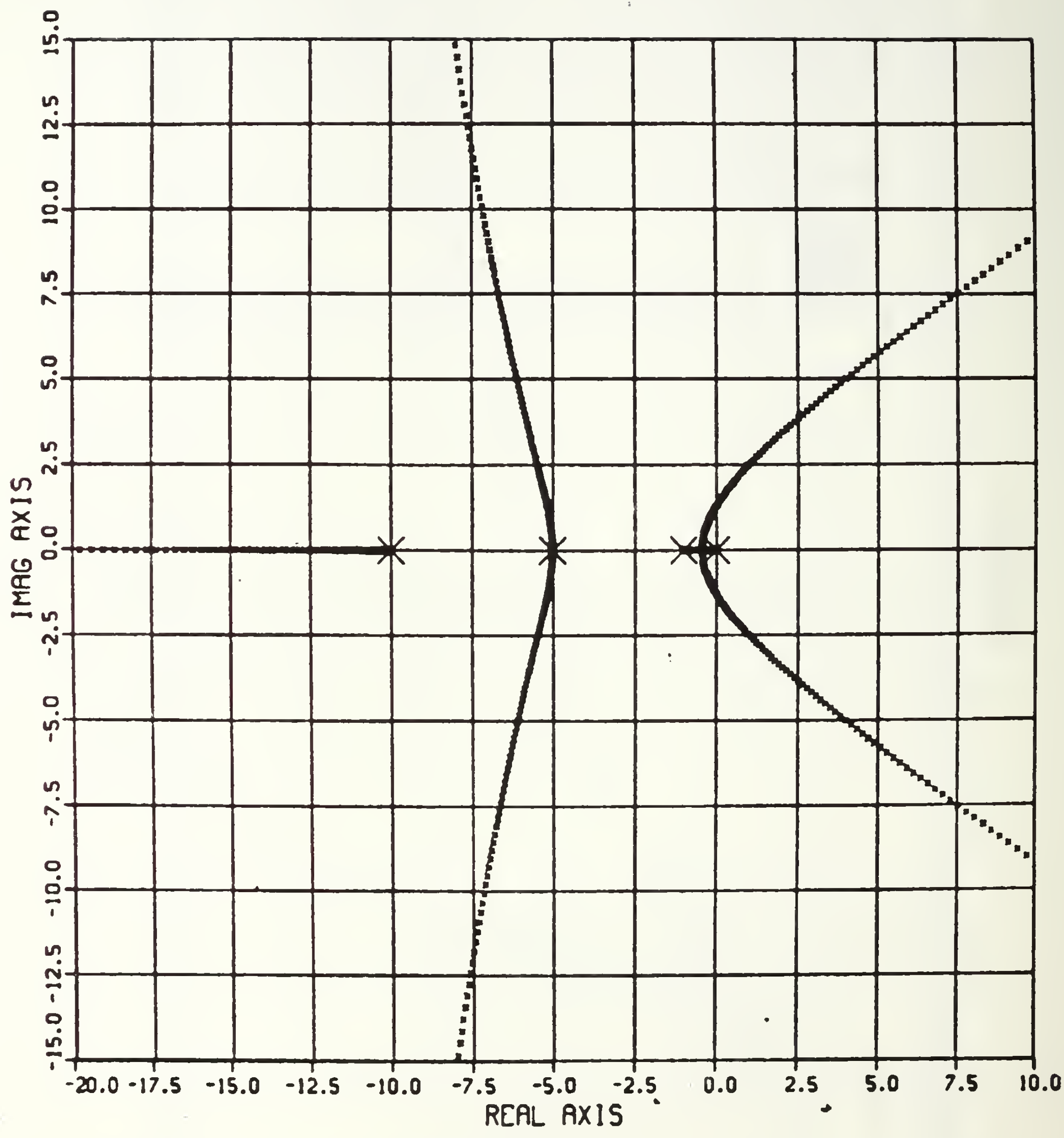

Figure 2.6.a Uncompensated System Root Locus Plot

roots to be chosen by the designer are placed at s= -4, -6 -9 and -11. The compensated root locus plot is shown in Figure 2.6.b. Figure 2.6.c shows acceptable zero offset locations at s= -5±j2. The G(s)H(s) function becomes

$$G(s)H(s)=\frac{k(s+5+2j)(s+5-2j)(s+4)(s+6)(s+9)(s+11)}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \qquad (2.12)$$

The compensated root locus plot with zero offset locations is shown in Figure 2.6.d. With a loop gain of 1,318, the dominant roots are located at s= -2±j2 as shown by Figure 2.6.e, and the system roots are located at

$$s= -2\pm j2,\ -4.75,\ -5.22,\ -9.98,\ -24.2,\ -1840.0 \qquad (2.13)$$

The compensated system block diagram is shown in Figure 2.6.f. The compensated system step response and BODE diagram are shown in Figures 2.6.g and 2.6.h respectively. If the step response shown in Figure 2.6.g does not meet the required time specifications of the system, the designer will need to pick a different set of dominant roots and redesign the problem.

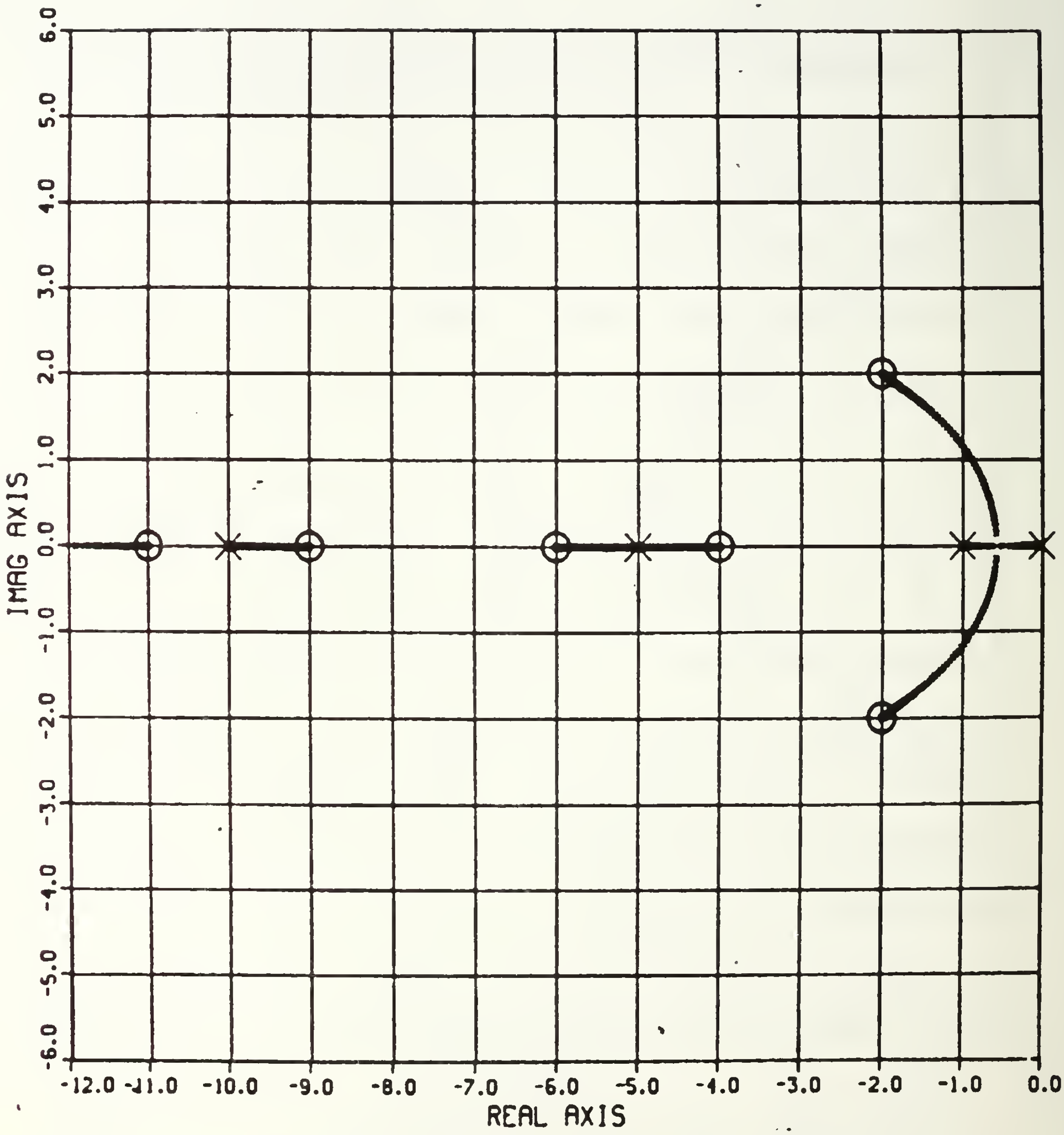

Figure 2.6.b Compensated System Root Loci without Zero Offsets

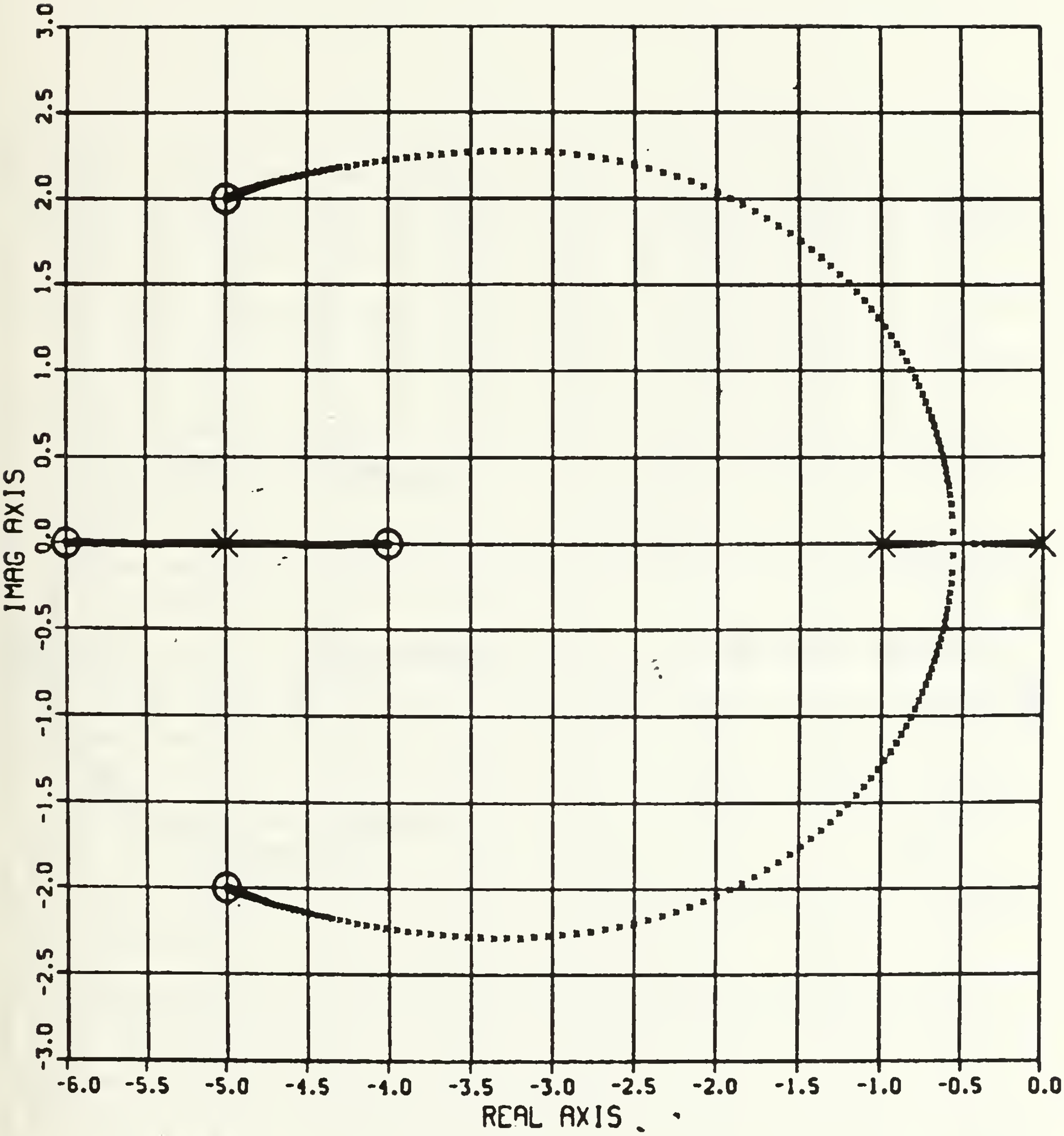

Figure 2.6.c Root Loci for Dominant Roots with Zero Offsets

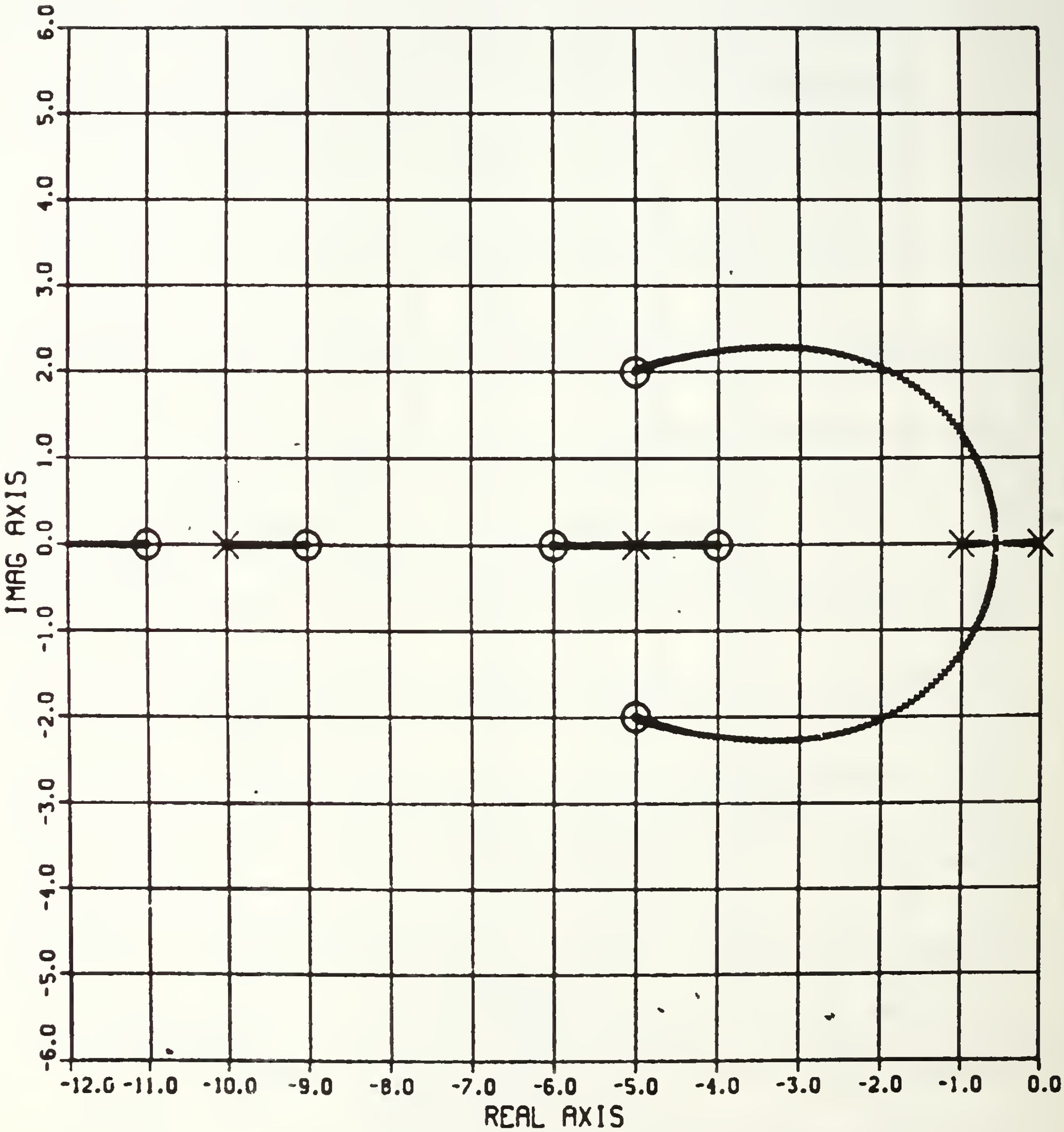

Figure 2.6.d Compensated System Root Loci with Zero Offsets

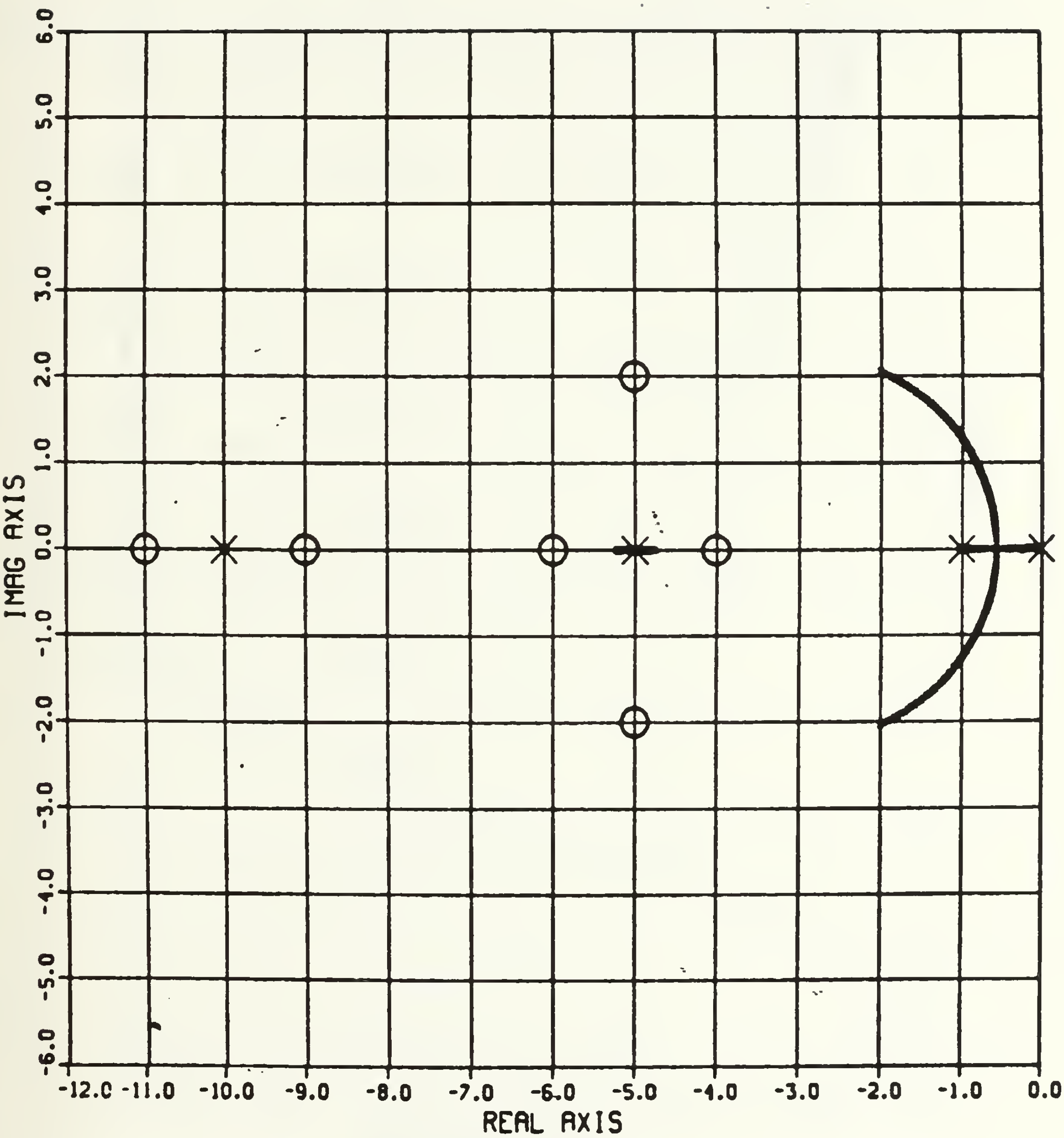

Figure 2.6.e Final Compensated System Root Locus Plot

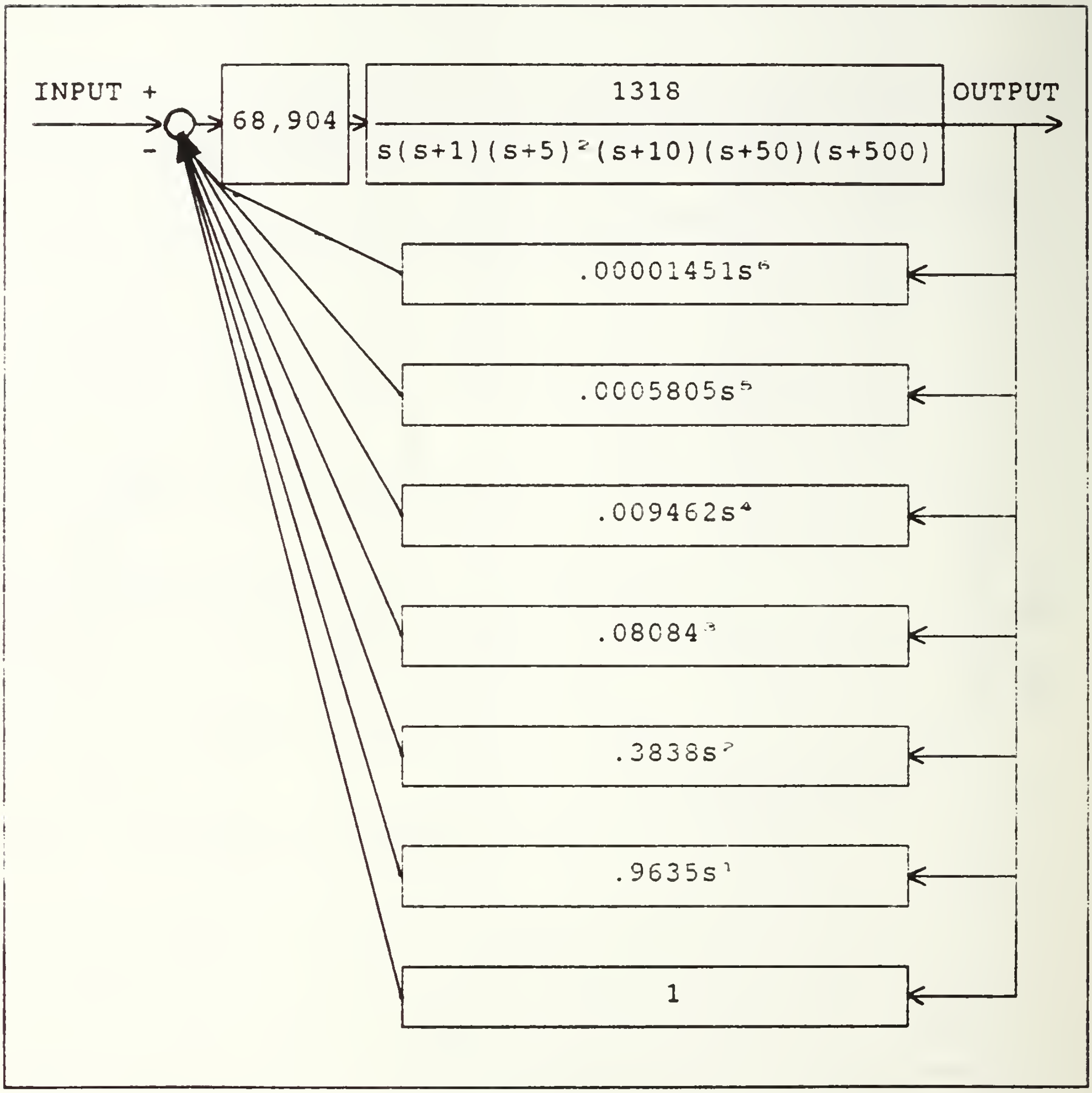

Figure 2.6.f Compensated System Block Diagram

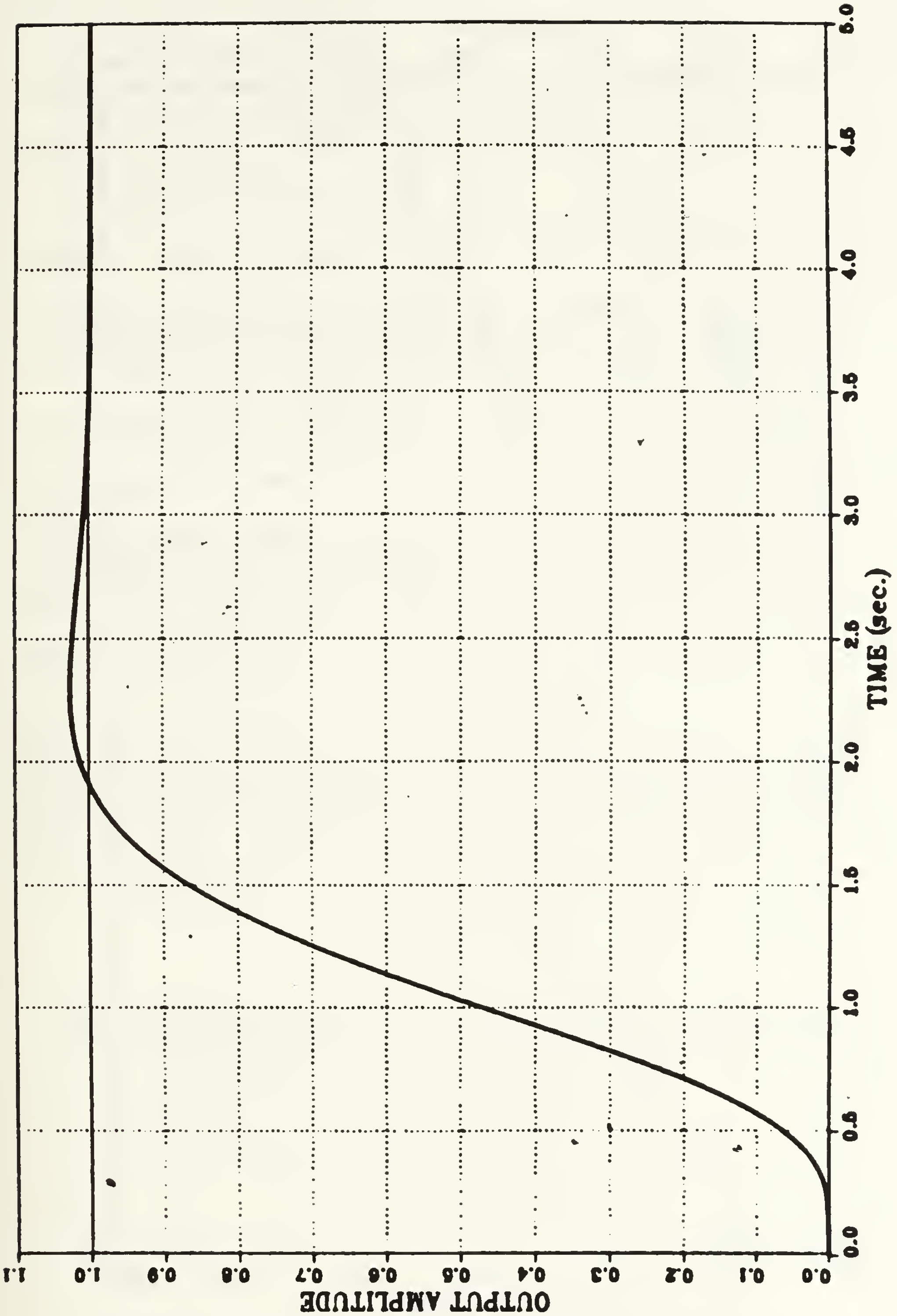

Figure 2.6.g Compensated System Step Response

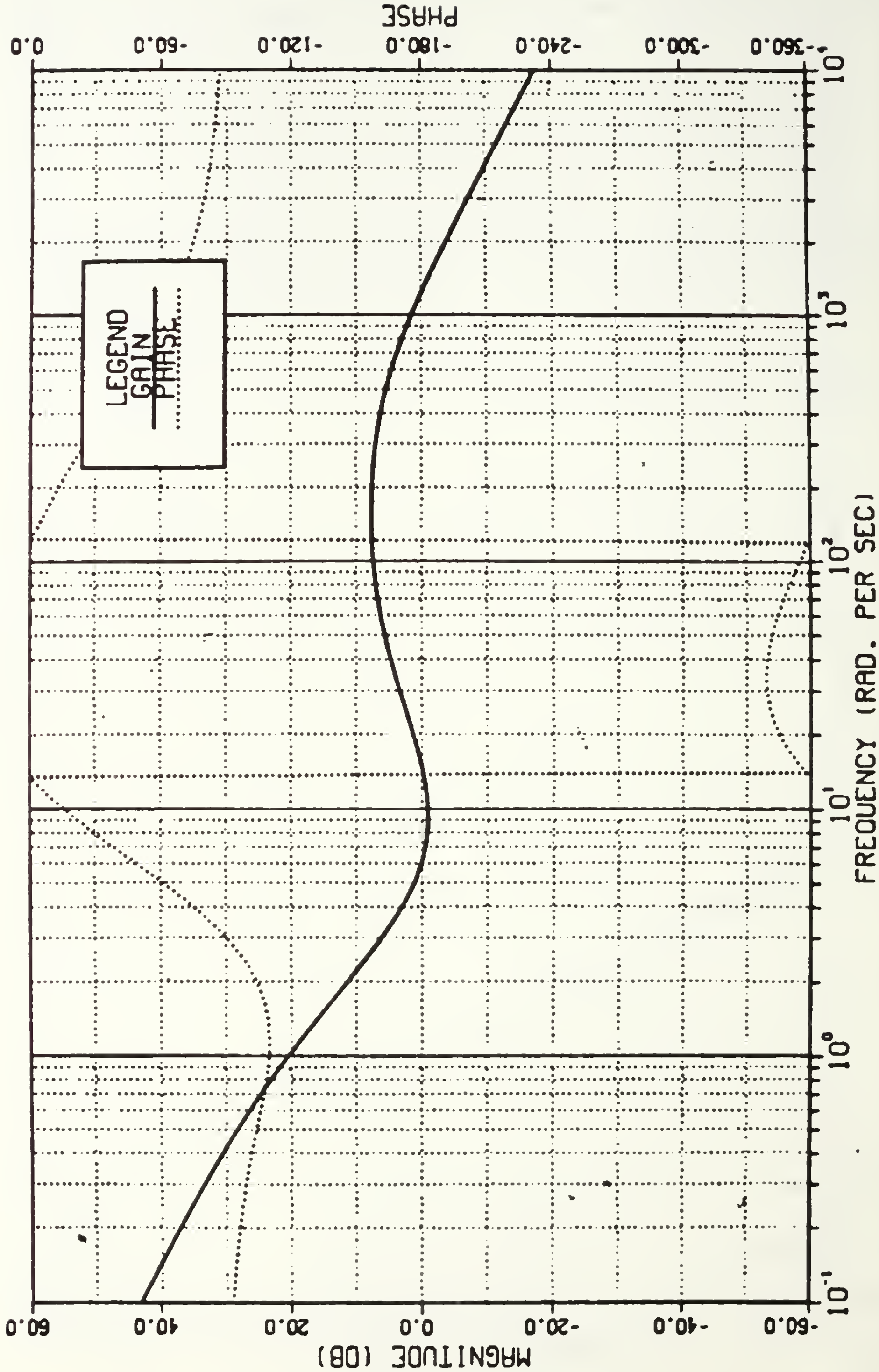

Figure 2.6.h Compensated System BODE Diagram

In this chapter a design procedure was presented for root placement with full state feedback using transfer function methods. With full state feedback, all N states are available to be measured and fedback. Therefore the feedback polynomial is of order N-1, and the system designer may choose locations for only N-1 roots. However, every root of the system may be specifically located by adjustment of the feedback coefficients. The N-1 roots specified by the designer are the zeros of the G(s)H(s) function. The system's dominant roots are chosen to meet the time specifications of the system. The system's unspecified root must be real and moves on the negative real axis towards infinity. The zeros that attract the system's dominant roots are offset to maintain the system error coefficient and to ensure a realizable system gain. If the required specifications of the system cannot be met using this design procedure, the designer should consider compensating this procedure with an alternate design scheme.

III. PARTIAL STATE FEEDBACK: ALL POLE PLANT

A. INTRODUCTION

In the last chapter we explored design procedures for root placement using full state feedback with all pole plants. All of the states had to be available to be fedback for this procedure. With partial state feedback, less than all of the states are available to be fedback. Clearly if the lowest ordered state is missing in the feedback path, there will be a root at or near the origin. In most systems this is not desirable. If intermediate states are missing in the feedback path, there will be zeros in the right half plane.[3] Therefore, it is assumed that only the higher ordered states are not available to be fedback.

It should be clear that the coefficient of the N-1 term in the characteristic equation is the summation of the closed loop roots. It is also the sum of the system's open loop poles [Ref. 1:p. 1]. With partial state feedback, this sum is fixed and cannot be adjusted. Knowing the coefficient of the N-1 term will give the system designer an initial indication of how much or how little flexibility is available in compensating the system to meet required specifications. In many engineering design cases, the time performance of the

[3] From the theory of equations described in virtually all classical control theory textbooks.

system is the critical design factor that must be satisfied. The time performance criteria of a system is usually measured as a function of the initial overshoot and number of oscillations to a given input, and the settling time of the transient response. Utilizing partial state feedback, roots are chosen in an attempt to meet required time performance criteria. There are plants that cannot be compensated to satisfy given system specifications using <u>only</u> partial state feedback. In such cases, a combination of partial state feedback and other compensation schemes should be considered by the engineer.

B. N-1 FEEDBACK STATES

When N-1 states are available to be measured and fedback, the system designer may choose locations for N-2 roots. The G(s)H(s) function will have two excess poles that will follow asymptotic angles of ±90°. These poles will attempt to go to infinity as the gain approaches infinity. If the breakaway point from the real axis of these two excess poles is far enough to the left of the dominant root locations, the system designer should not experience much difficulty in designing the system to meet given specifications.

EXAMPLE 3.1 Equation 3.1 defines the plant for a fourth order system.

$$G(s) = \frac{K}{s(s+5)^2(s+10)} \tag{3.1}$$

Dominant root locations have been chosen at s= $-2\pm j2$ to satisfy required system time performance and bandwidth specifications. Note that with a fourth order system, the designer may only choose the dominant root locations! The uncompensated root locus plot is shown in Figure 3.1.a, and the compensated root locus plot is shown in Figure 3.1.b. The summation of the open loop poles is 20, and the numerical value for the sum of the two specified dominant roots is 4. Therefore a numerical value of 16 remains for the sum of the two unspecified roots that will follow asymptotic paths of $\pm 90°$ to infinity as the gain approaches infinity. Consequently, the two unspecified roots should breakaway from the real axis at approximately s= -8.0. This is confirmed by inspection of Figure 3.1.b. An extremely high gain is required to move the roots from their open loop pole locations to the desired root locations. By using the zero offset technique, it is possible to reduce the system gain and attempt to maintain a reasonable error coefficient. By trial and error, zero offset locations are determined at s= $-3.2\pm j3.0$ such that the root loci pass through the dominant root locations at s= $-2\pm j2$. The root locus plot utilizing the zero offset locations for the dominant roots is shown in Figure 3.1.c. With a root locus gain of 37.3, the system's dominant roots are located at s= $-2\pm j2$ as depicted in Figure 3.1.d. The compensated system block diagram is shown in Figure 3.1.e.

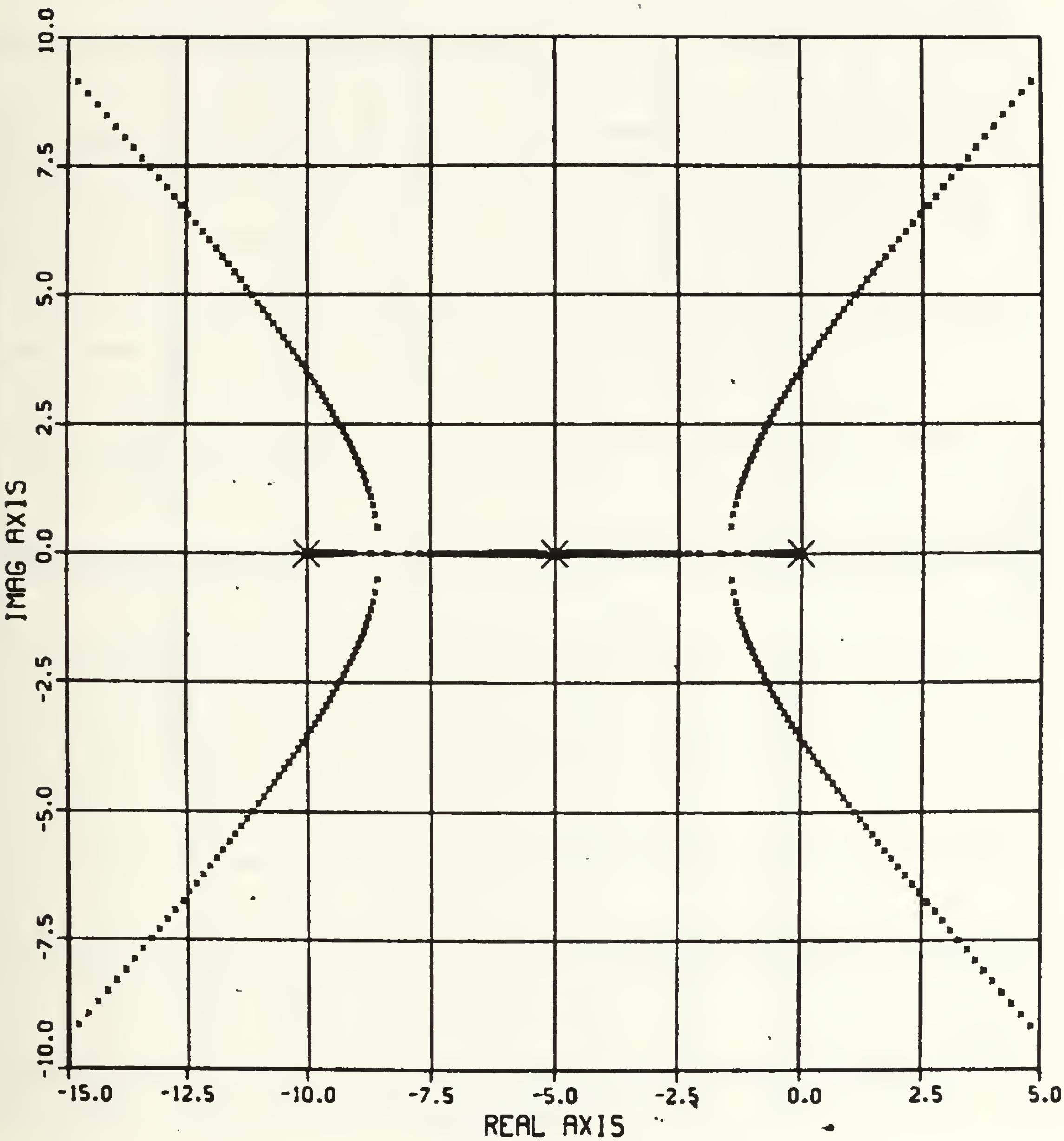

Figure 3.1.a Uncompensated System Root Locus Plot

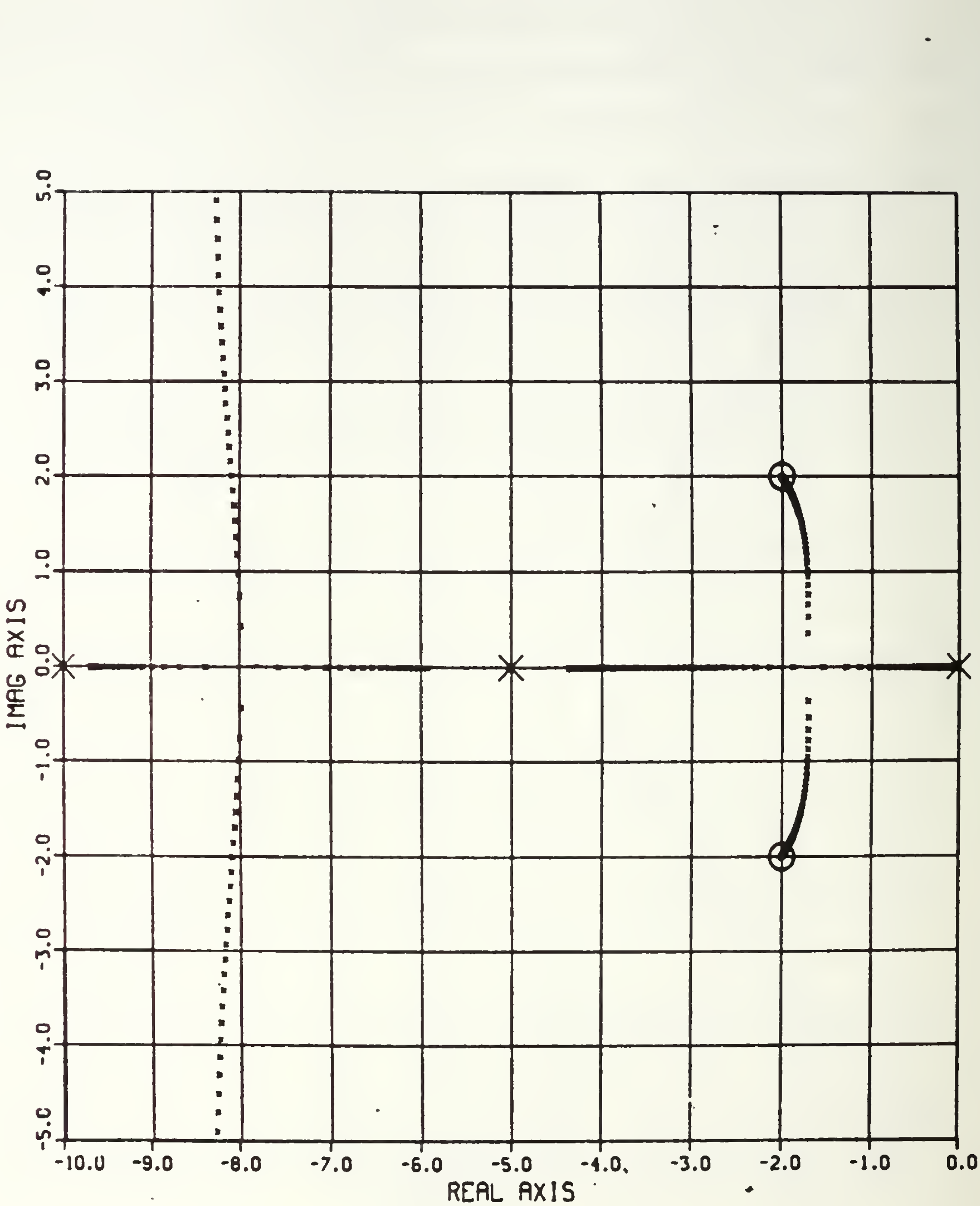

Figure 3.1.b Compensated System Root Loci without Zero Offsets

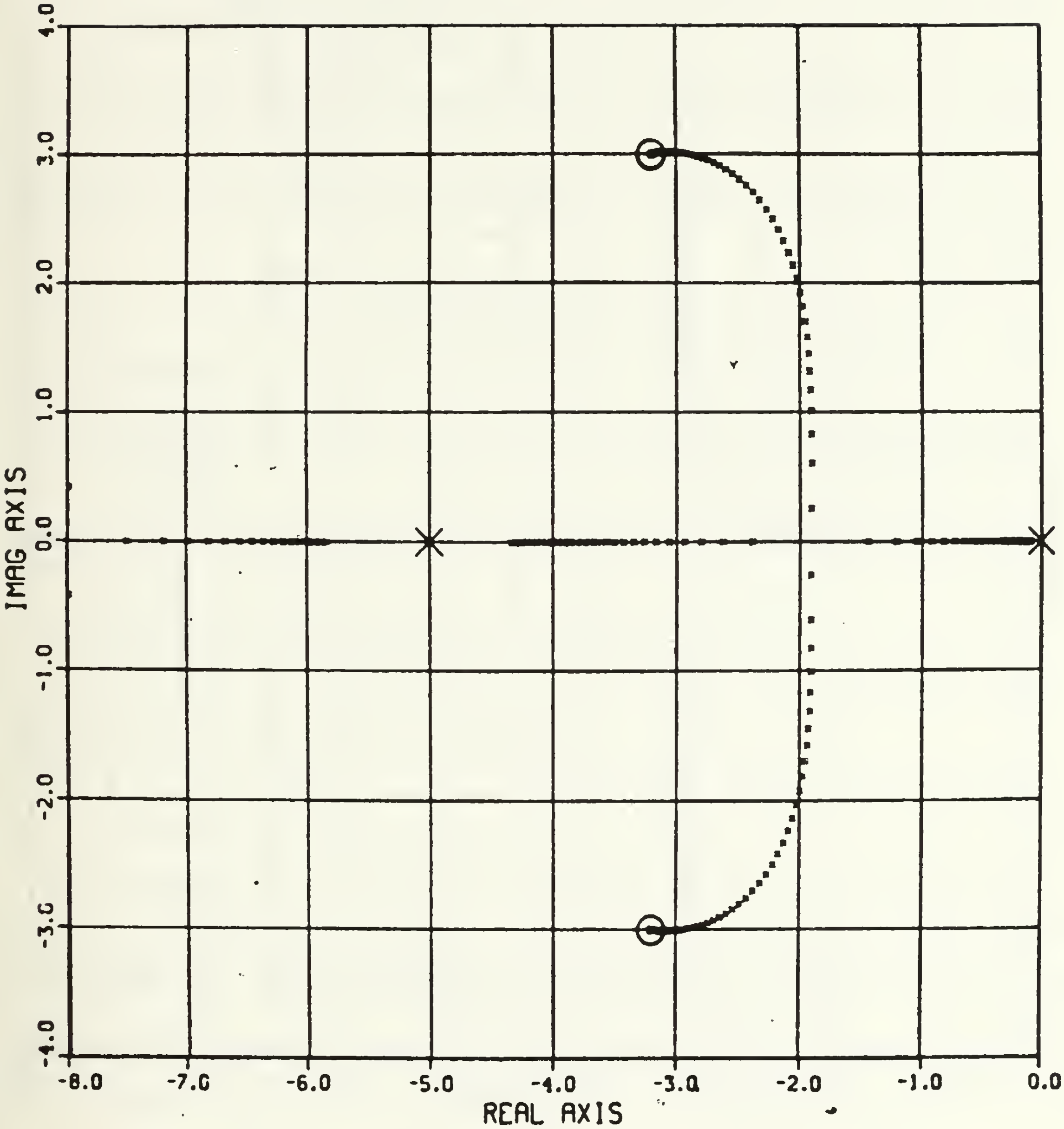

Figure 3.1.c Compensated System Root Loci with Zero Offsets

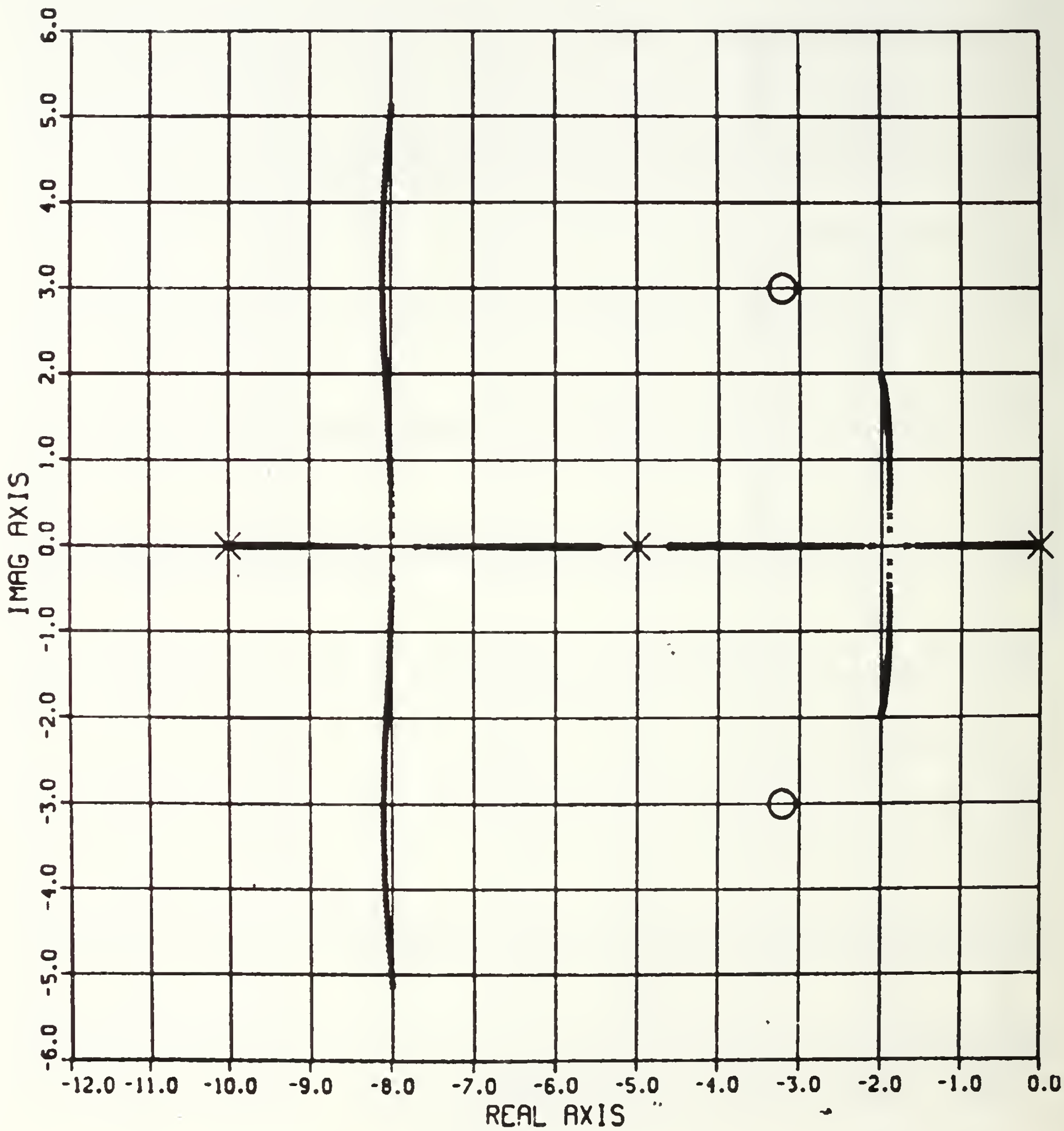

Figure 3.1.d Final Compensated System Root Locus Plot

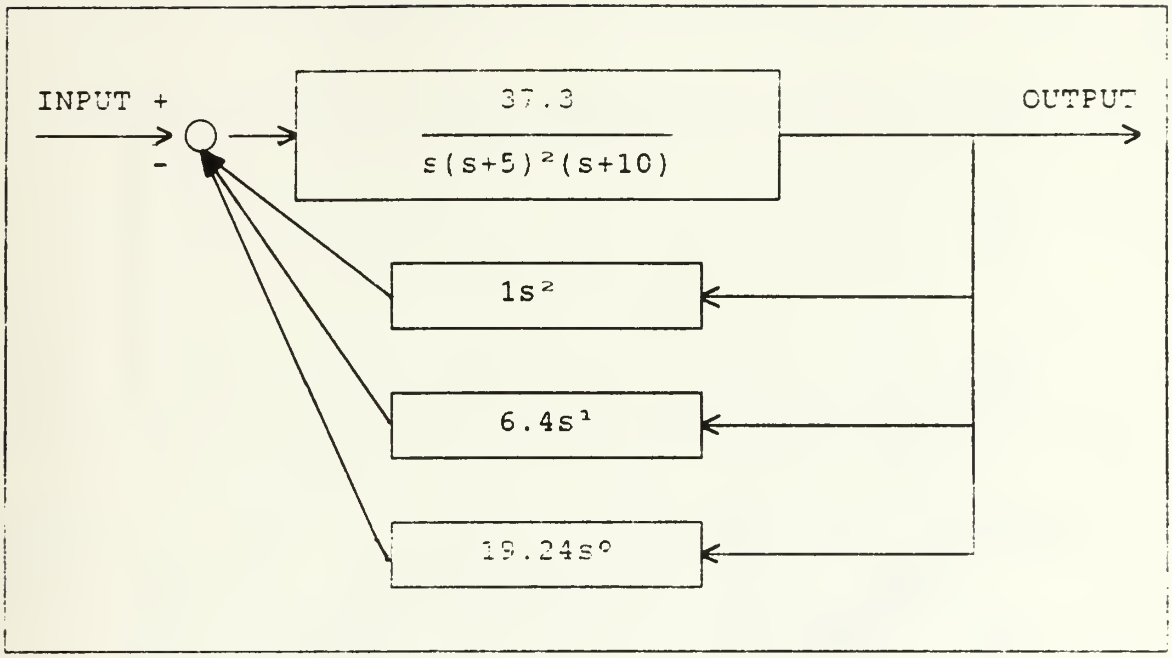

Figure 3.1.e Compensated System Block Diagram

The system characteristic equation is

$$s^4+20.s^3+162.3s^2+488.7s^1+717.7=0 \qquad (3.2)$$

which may be factored to yield

$$(s+8\pm j5.13)(s+2\pm j1.98)=0 \qquad (3.3)$$

To preserve unity feedback, the gain of the s^0 term may be relocated in the forward path as shown in Figure 3.1.f. The compensated system step response and BODE diagram are shown in Figure 3.1.g and Figure 3.1.h respectively. If the resultant step response does not satisfy the required system time performance criteria, the designer must choose a different set of dominant roots and redesign, or consider an alternate compensation scheme.

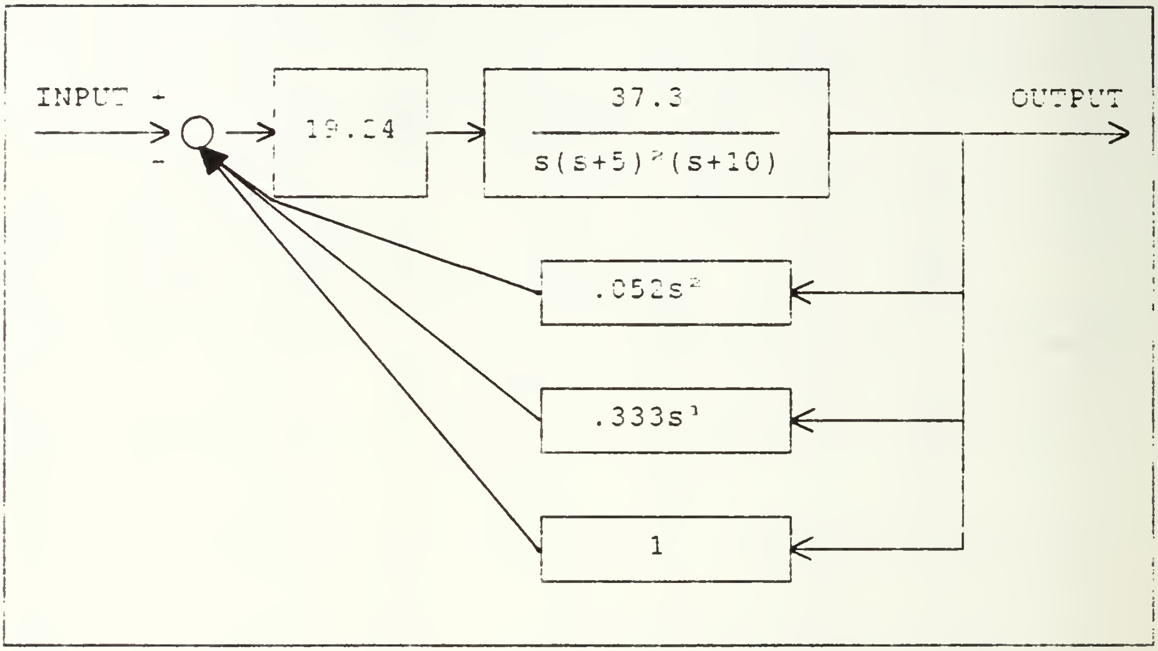

Figure 3.1.f Compensated System Block Diagram

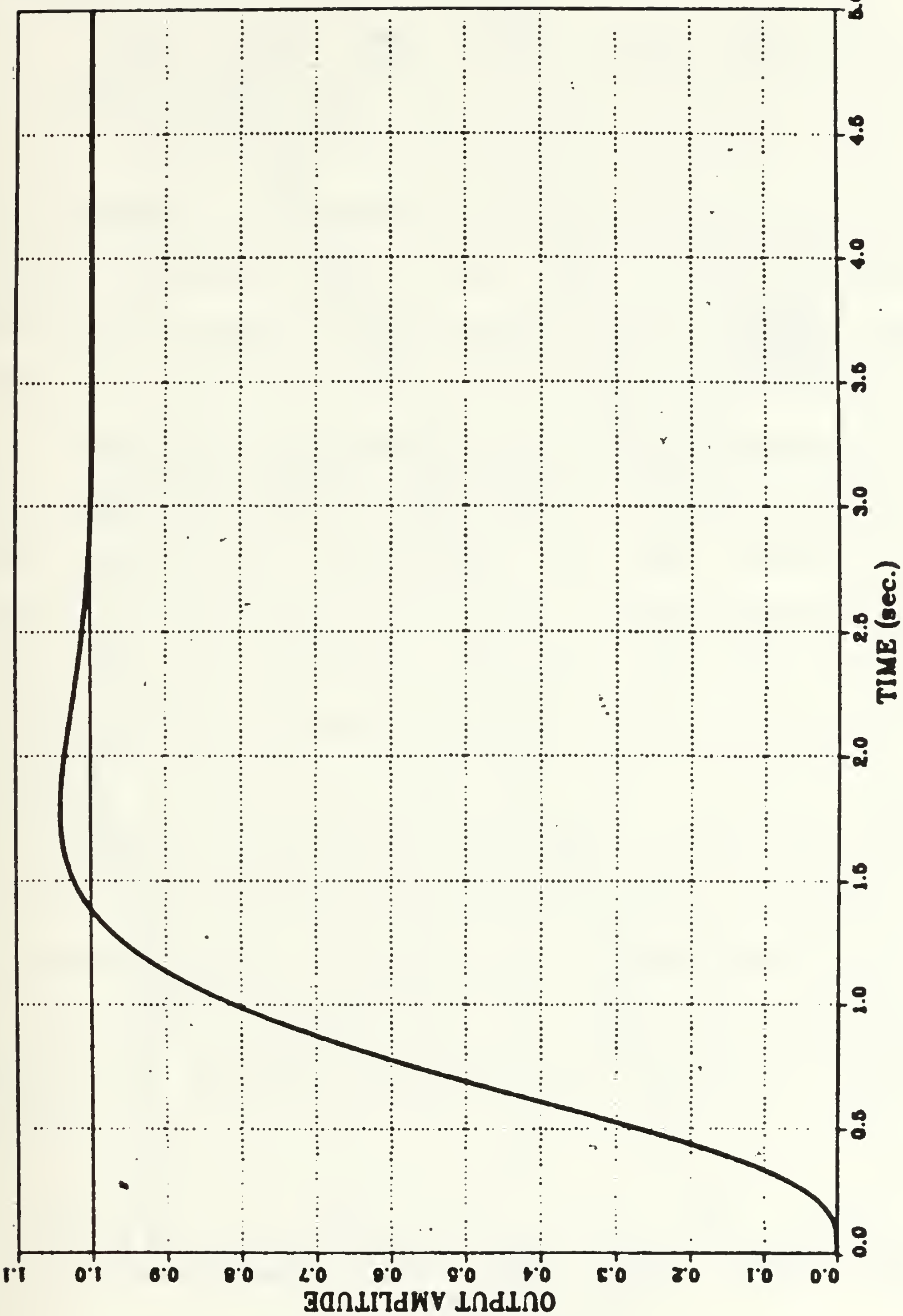

Figure 3.1.g Compensated System Step Response

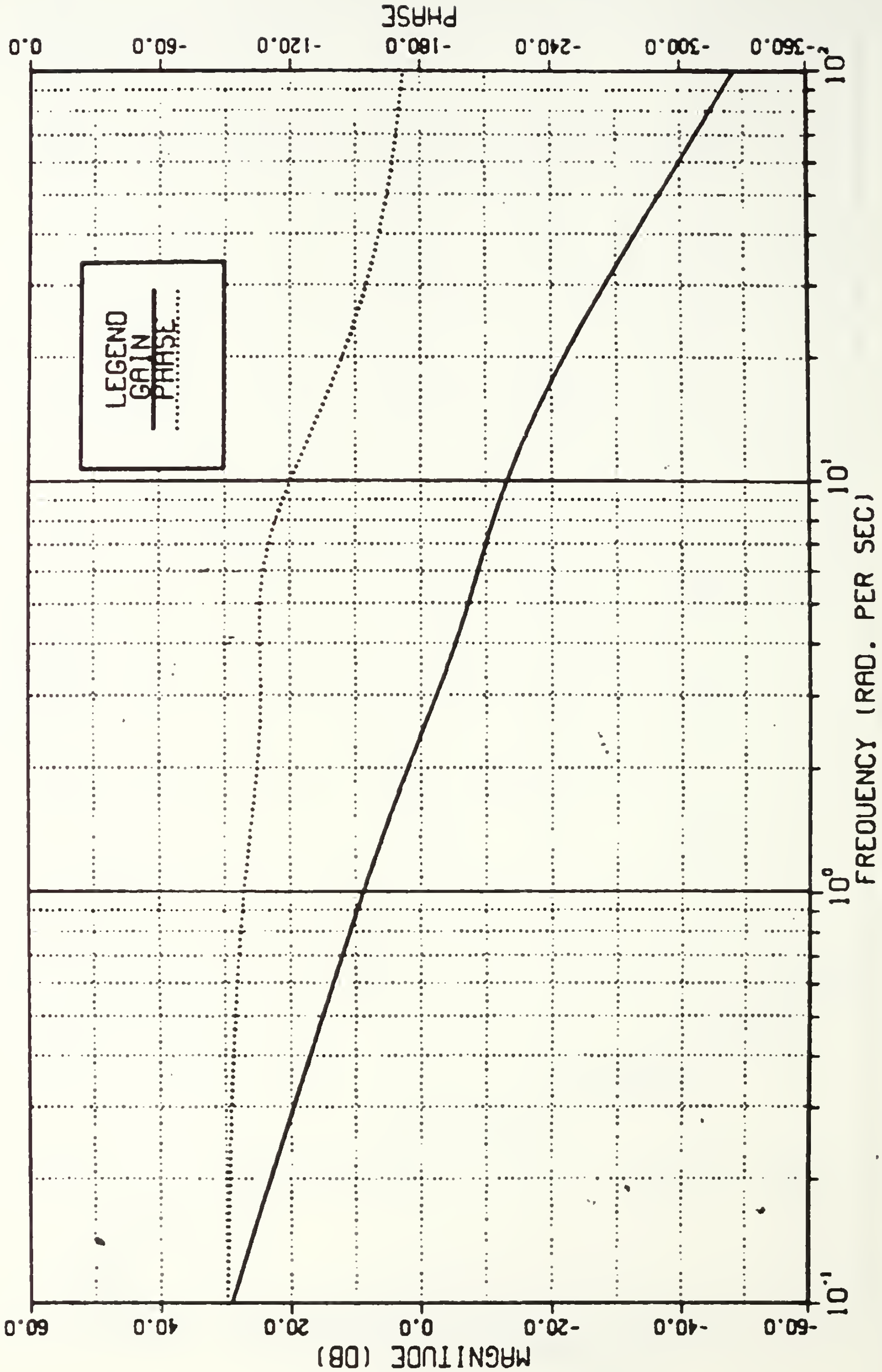

Figure 3.1.h Compensated System BODE Diagram

EXAMPLE 3.2 The plant for a different fourth order system is defined by equation 3.4.

$$G(s)=\frac{K}{s(s+1)^2(s+5)} \tag{3.4}$$

As in example 3.1, dominant roots are chosen at s= -2±j2 to satisfy required system time performance specifications. With this particular plant however, the sum of the open loop poles is 7, and the sum of the two specified dominant roots is 4. Therefore the numerical sum of the real parts of the two unspecified roots <u>must</u> equal 3. Using classical root locus evaluation techniques, the two unspecified root locations must lie to the right of the specified dominant root locations. The sum of the open loop poles must equal the sum of the closed loop system roots using partial state feedback. It should be clear that with this particular all pole plant it is not possible to meet the required specifications using only partial state feedback. The uncompensated root locus plot and the compensated root locus plot are shown in Figure 3.2.a and Figure 3.2.b respectively.

EXAMPLE 3.3 Equation 3.5 defines the plant for a sixth order system.

$$G(s)=\frac{K}{s(s+1)(s+5)^2(s+10)(s+50)} \tag{3.5}$$

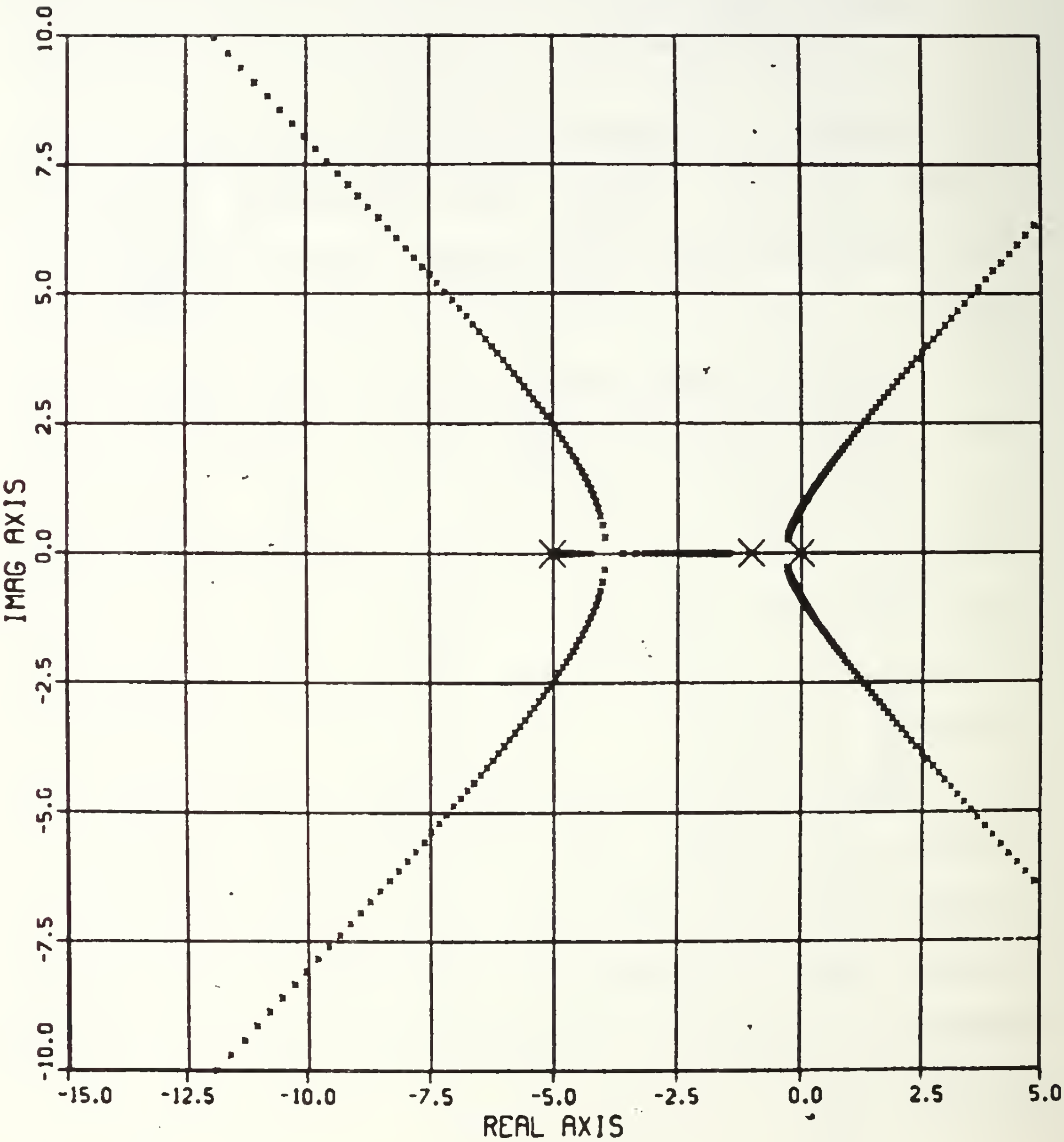

Figure 3.2.a Uncompensated System Root Locus Plot

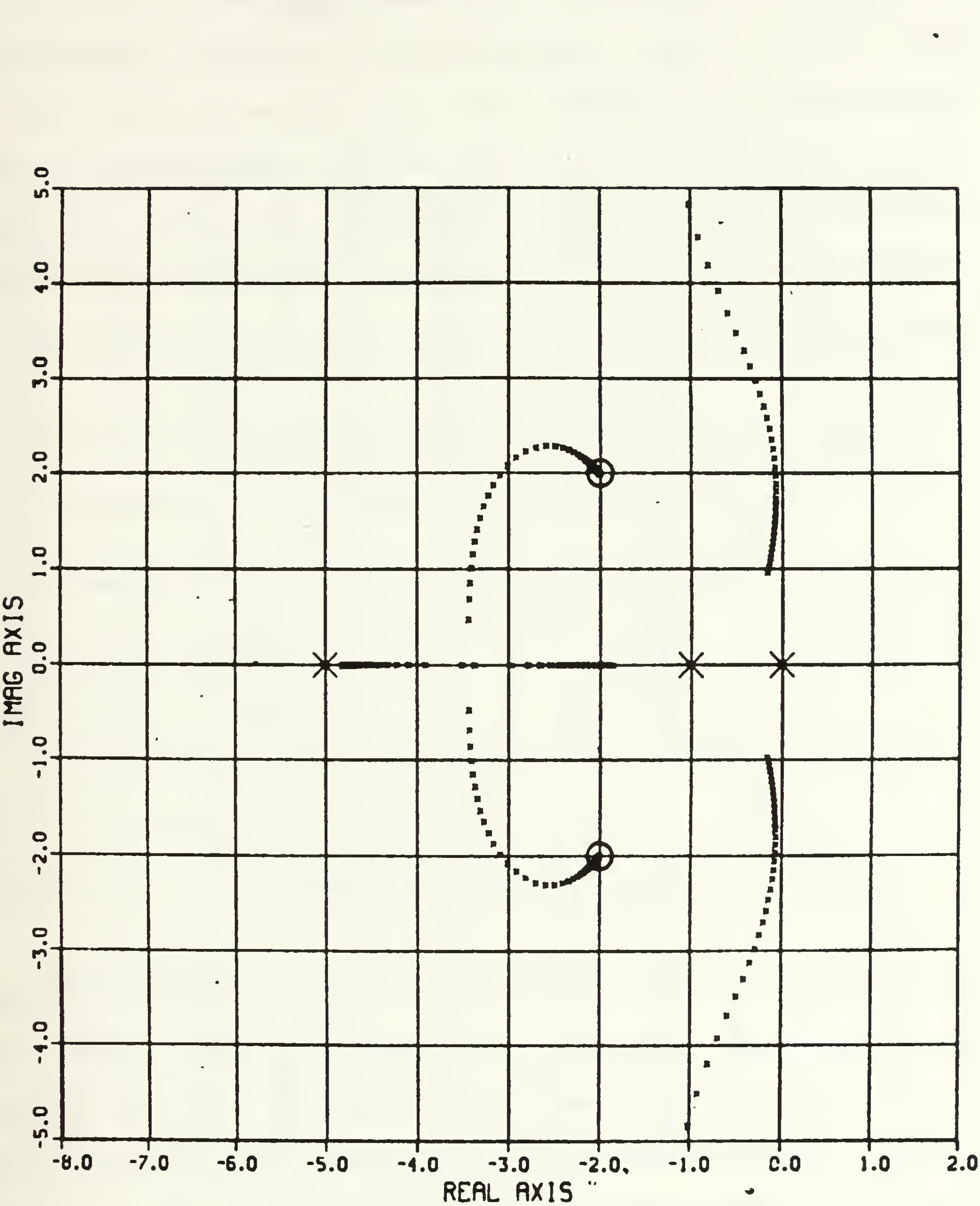

Figure 3.2.b Compensated System Root Locus Plot

Once again the dominant roots are required at $s=-2\pm j2$ to meet given system specifications. As the uncompensated system gain is increased the roots follow the loci shown in Figure 3.3.a. The root loci of the uncompensated system cross the jw axis with a gain equal to 27,542. The plant transfer function can be rewritten in BODE form as shown in equation 3.6

$$G(s)=\frac{K}{12{,}500(s)(s+1)\left(\frac{s}{5}\;1\right)^{2}\left(\frac{s}{10}+1\right)\left(\frac{s}{50}+1\right)} \tag{3.6}$$

which produces an error coefficient $K_v=\frac{K}{12{,}500}$.

Four roots may be chosen by the designer. The $G(s)H(s)$ function becomes

$$G(s)H(s)=\frac{K(s+2+j2)(s+2-j2)(s+r_3)(s+r_4)}{s(s+1)(s+5)(s+5)(s+10)(s+50)} \tag{3.7}$$

The dominant roots have been chosen to satisfy the time performance and bandwidth requirements of the system. The system designer must now determine where to place roots r_3 and r_4. The remaining two specified roots allowed to be chosen by the designer should be chosen such that the roots move as little as possible from their natural open loop pole locations yet allow the dominant roots to retain their dominant system role. In this particular example, roots r_3

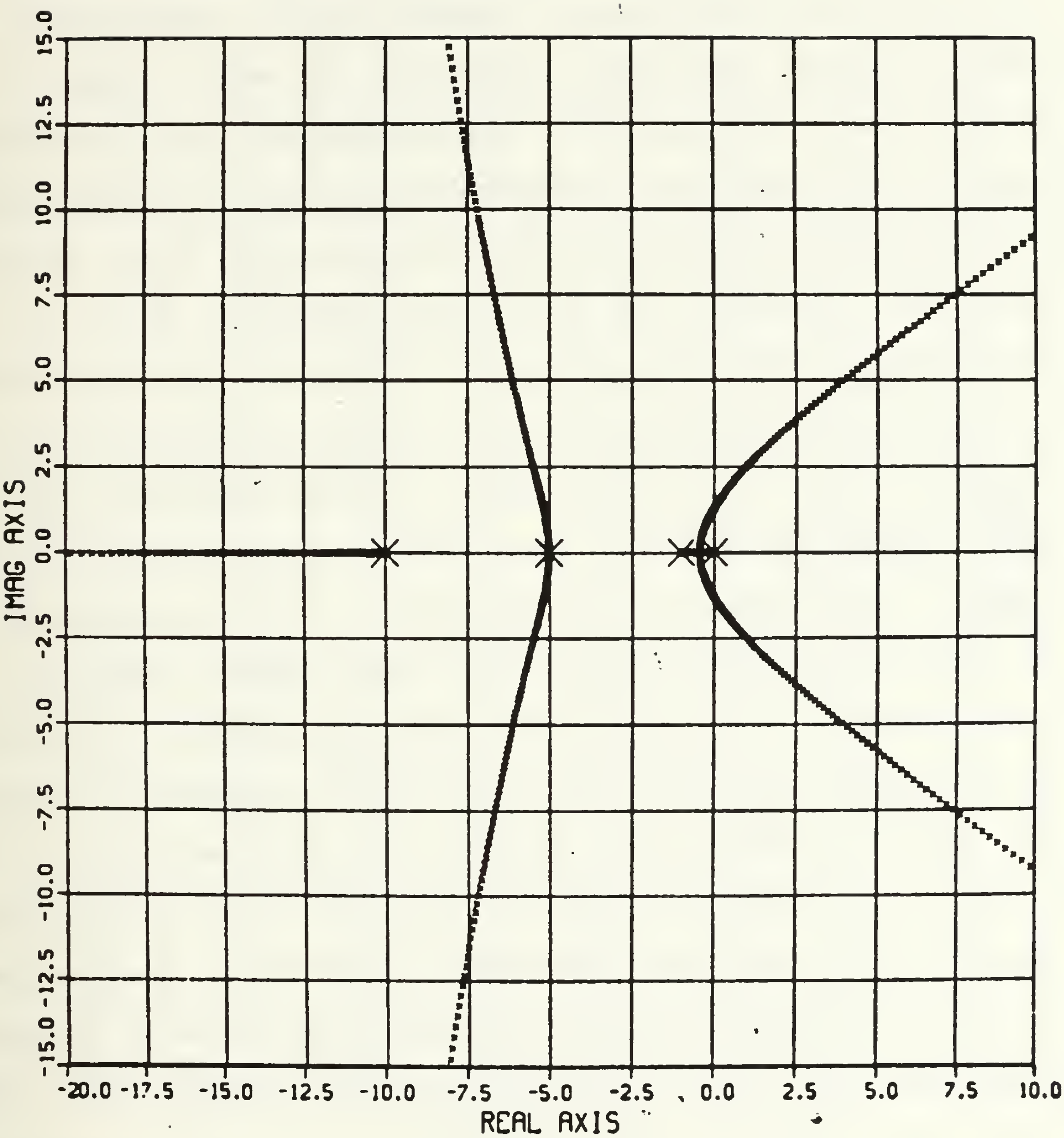

Figure 3.3.a Uncompensated System Root Locus Plot

and r_4 will be chosen at s= -4 and s= -6. This choice will not move the open loop roots from their natural locations at s= -5 and s= -5 significantly, and the dominant roots at s= -2±j2 will retain their dominant role. Furthermore, we want to control the roots originating at s= -5 and s= -5 because they are nearest the desired dominant root locations. Figure 3.3.b shows the compensated system root locus plot. The numerical summation of the open loop poles is 71, and the numerical summation of the four specified roots is 14. Therefore a numerical value of 57 remains for the sum of the two remaining unspecified roots. This indicates that the remaining two unspecified roots should breakaway from the real axis at approximately s= -28.5, significantly to the left of the dominant root locations. This is confirmed by inspection of Figure 3.3.b. The zero offset technique is used to reduce the root locus gain and attempt to maintain the original open loop error coefficient. Figure 3.3.c shows the general shape of the dominant root loci as the zeros of the G(s)H(s) function approach the poles of the G(s)H(s) function as the gain approaches infinity. By trial and error, zero offset locations are determined to be at s= -4±j1 such that the root loci pass through s= -2±j2. The root locus plot with zero offset locations at s= -4±j1 is shown in Figure 3.3.d. With a root locus gain of 325, the system's dominant roots are located at s= -2±j2 as depicted in Figure 3.3.e. Note that the roots other than the dominant roots

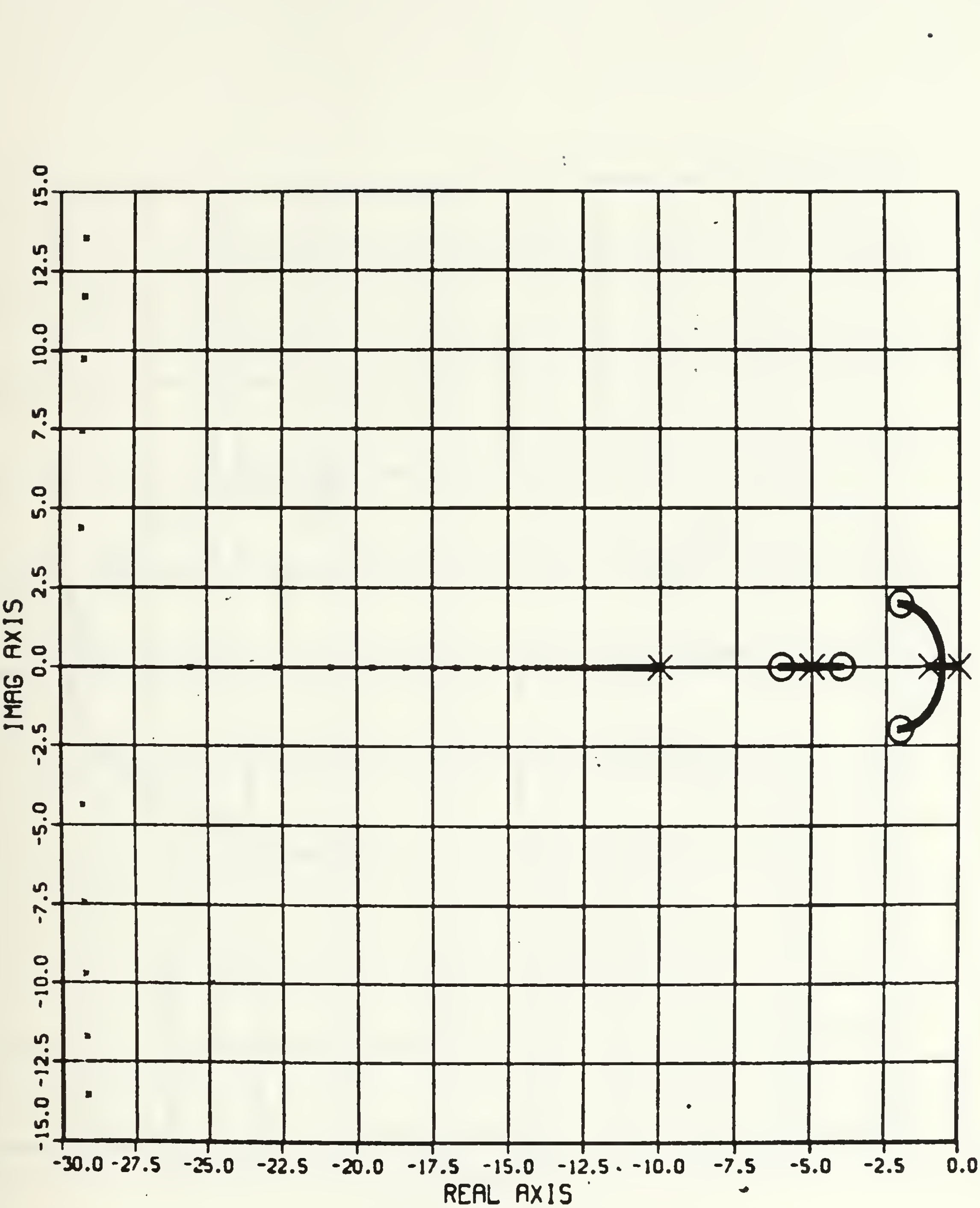

Figure 3.3.b Compensated System Root Loci without Zero Offsets

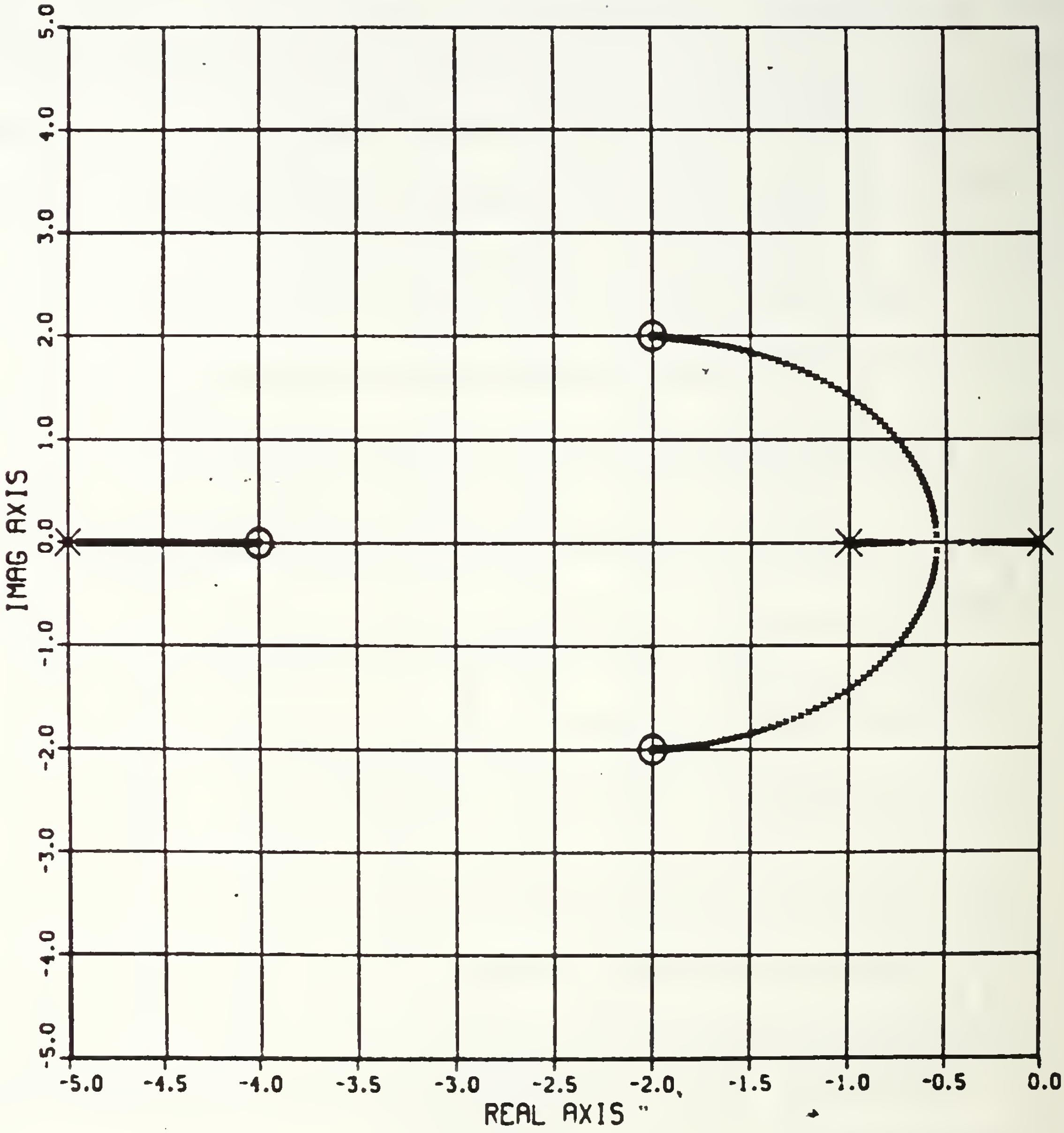

Figure 3.3.c Root Loci for Dominant Roots without Zero Offsets

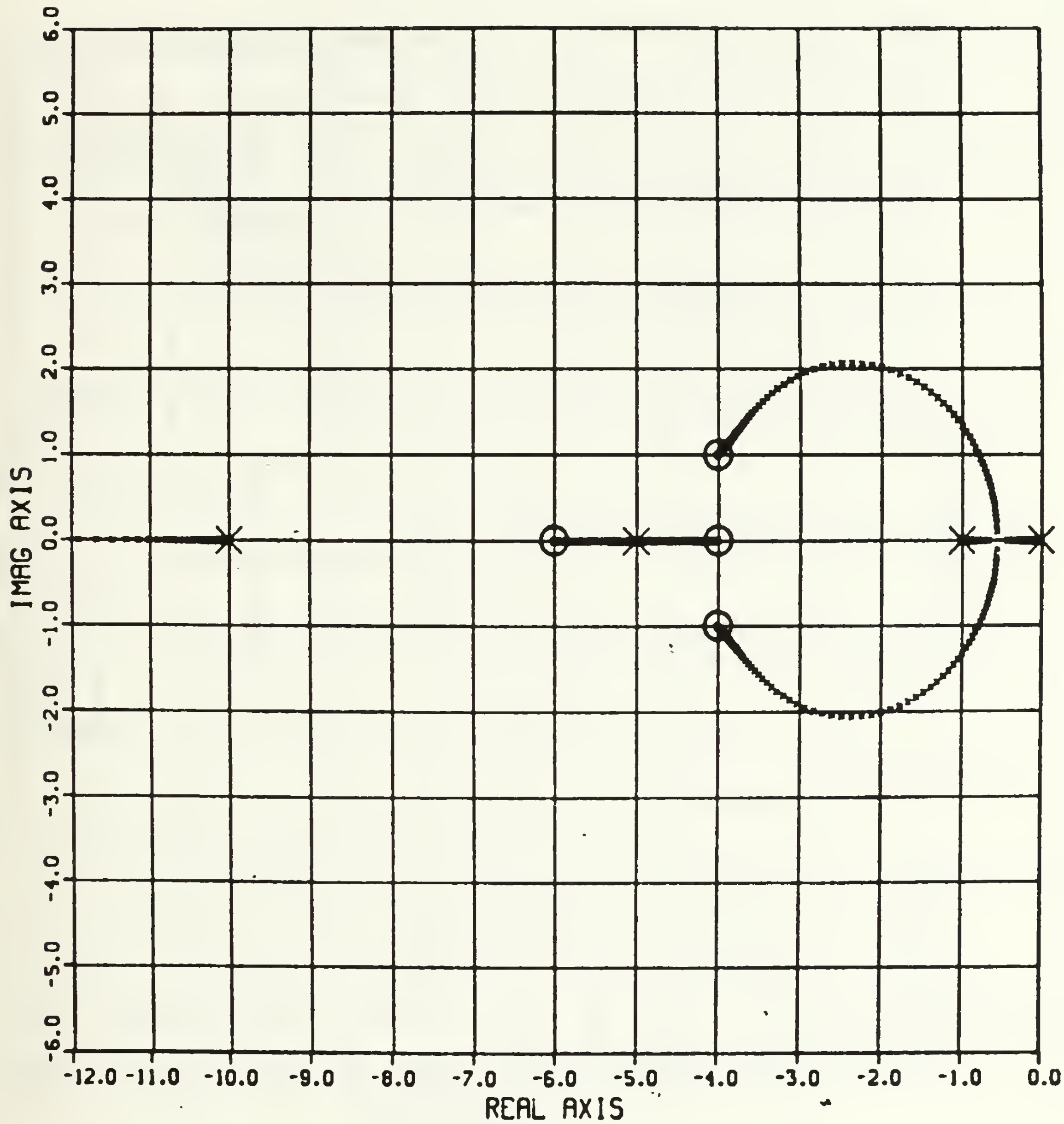

Figure 3.3.d Compensated System Root Loci with Zero Offsets

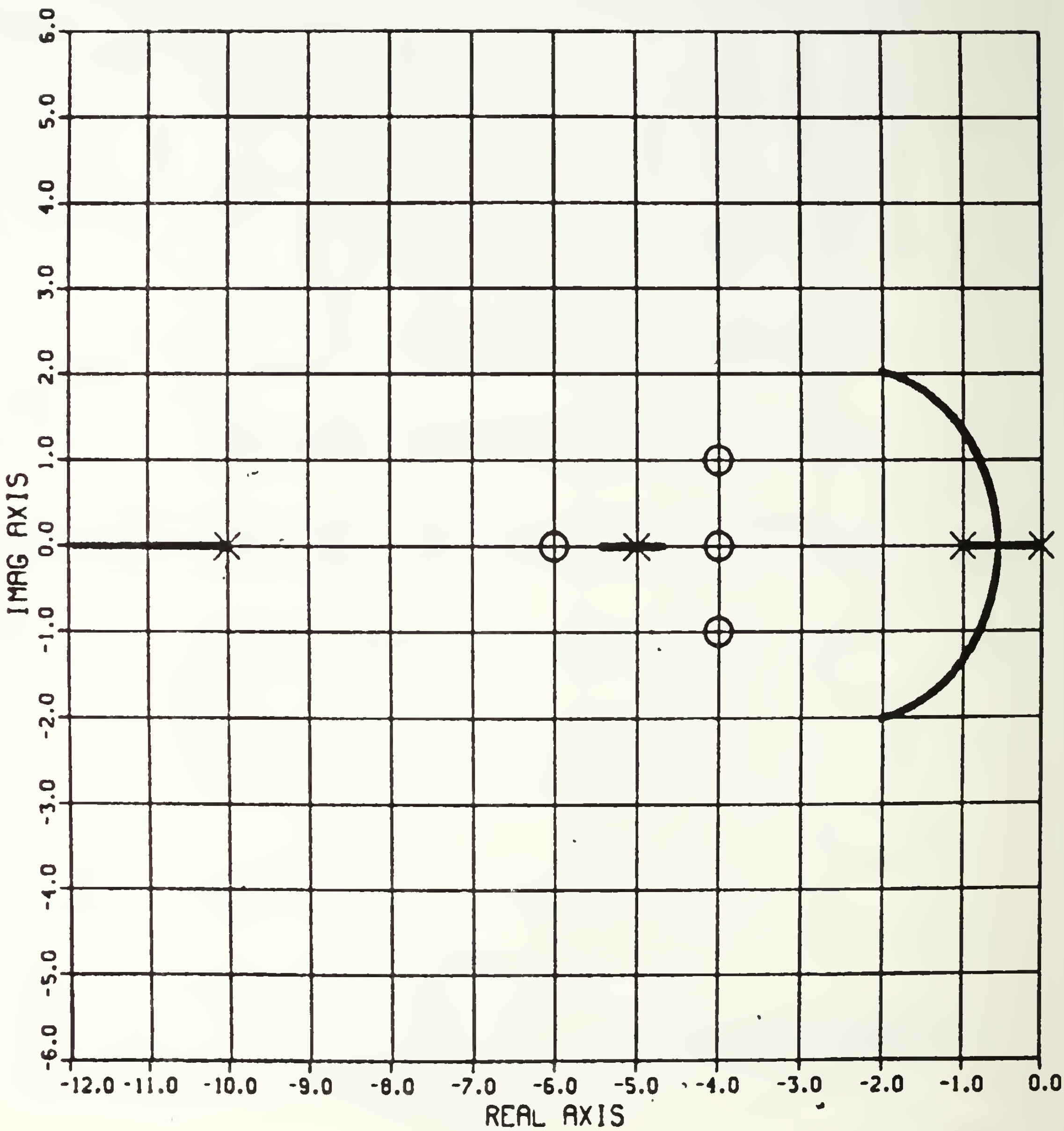

Figure 3.3.e Final Compensated System Root Locus Plot

have not been significantly moved from their natural open loop pole locations. The compensated system block diagram is shown in Figure 3.3.f.

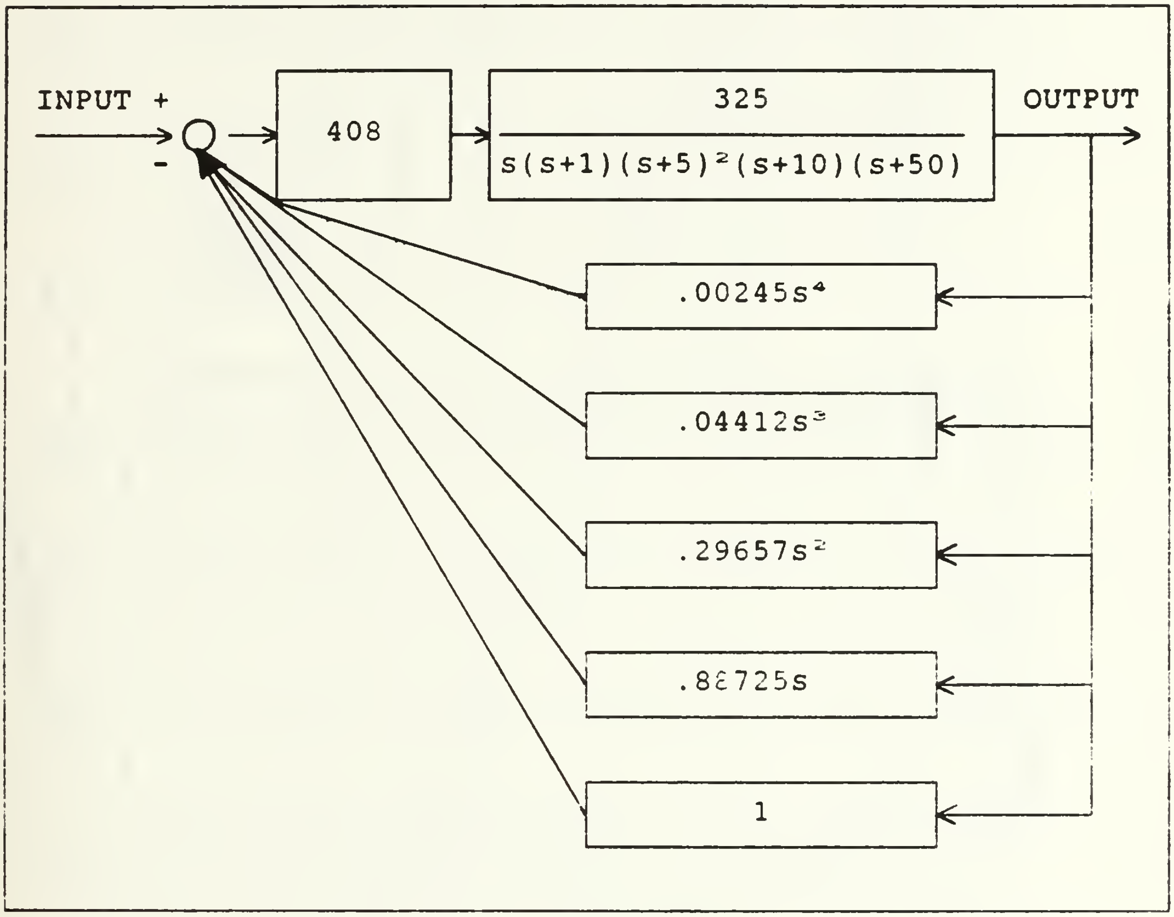

Figure 3.3.f Compensated System Block Diagram

The compensated system step response and BODE diagram are shown in Figures 3.3.g and 3.3.h respectively.

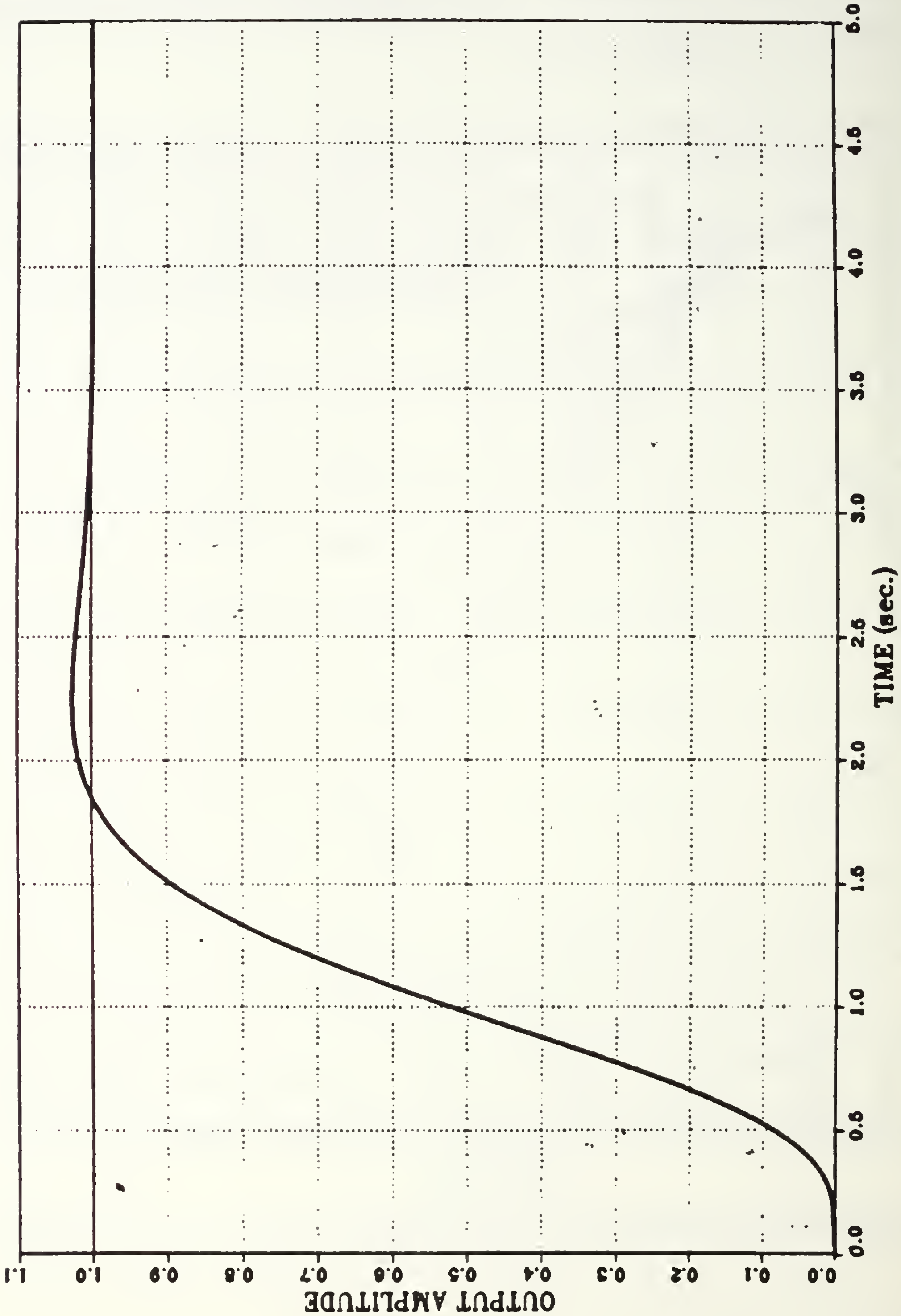

Figure 3.3.g Compensated System Step Response

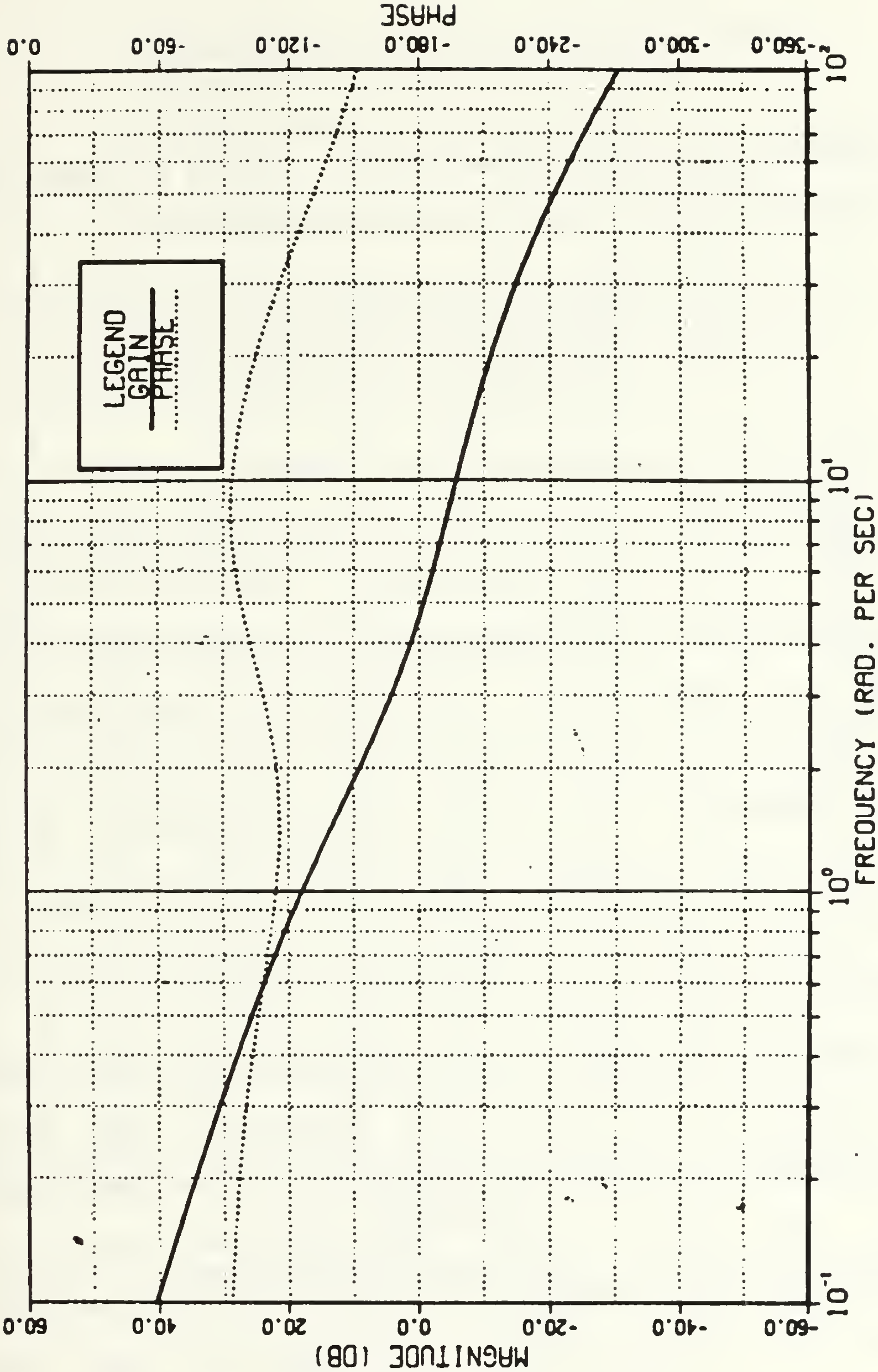

Figure 3.3.h Compensated System BODE Diagram

EXAMPLE 3.4

$$G(s) = \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \tag{3.8}$$

Dominant roots are chosen at s= -2±j2 to meet given system specifications. The G(s)H(s) function is

$$G(s)H(s) = \frac{K(s+2+j2)(s+2-j2)(s+r_3)(s+r_4)(s+r_5)}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \tag{3.9}$$

Roots r_3, r_4 and r_5 are chosen at s= -4, -6 and -9 respectively. These root locations are chosen such that the roots do not move significantly from their natural open loop pole locations, and the chosen dominant roots retain their dominant system role. It should be clear that those roots originating at poles farthest to the left of the desired dominant root locations are not controlled by the designer's chosen root locations. The farther away an open loop pole is from the origin, the more gain is required to move that root from its natural open loop pole location. By controlling those roots closest to the dominant roots, the concept of not moving the roots more than necessary from their natural location is reinforced. The sum of the open loop poles is 571, and the sum of the five specified roots is 23. The remaining two unspecified roots should breakaway from the real axis significantly to the left of the dominant root locations. Figure 3.4.a shows the compensated root locus

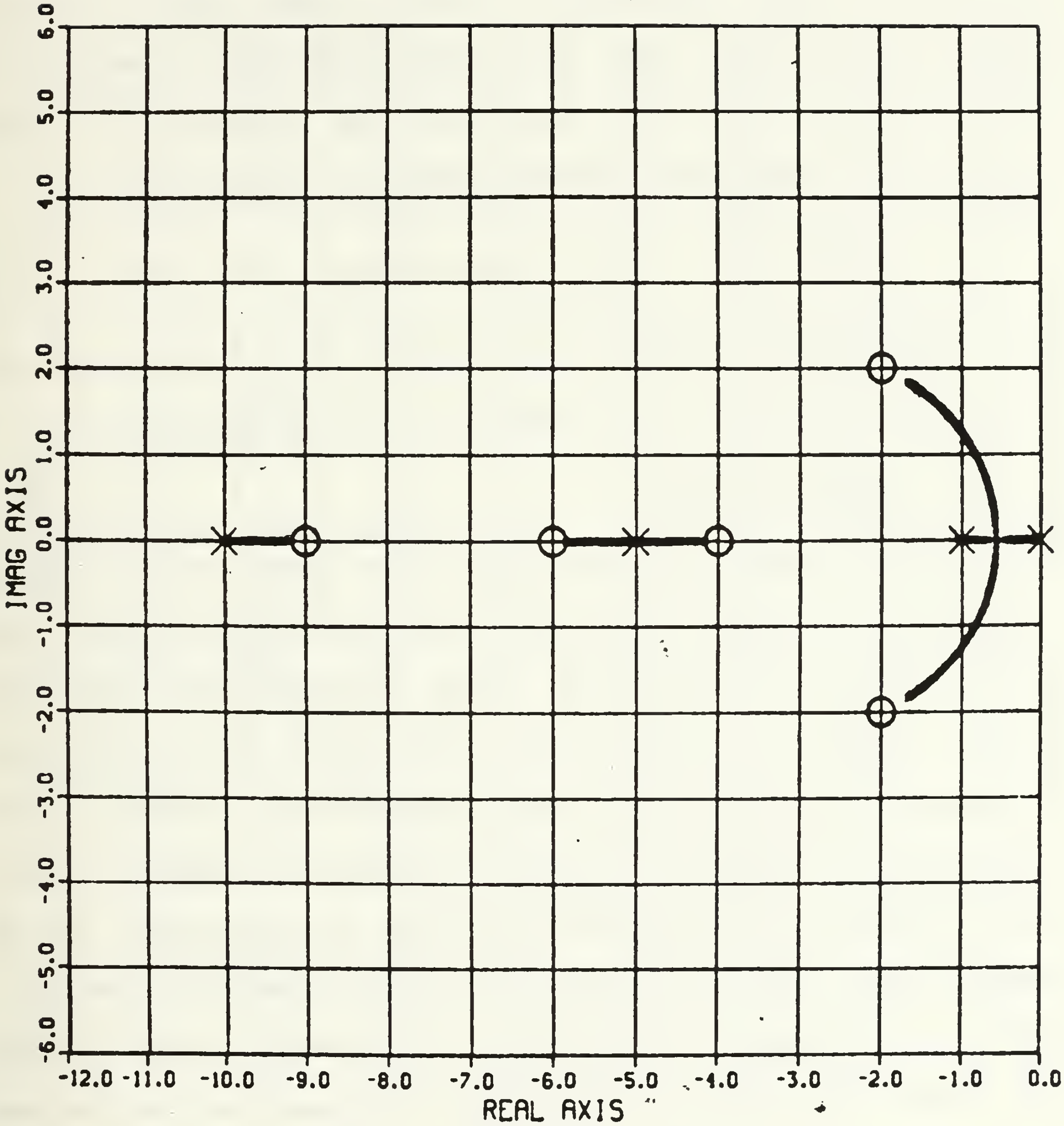

Figure 3.4.a Compensated System Root Loci without Zero Offsets

plot as the gain approaches infinity. Zero offset locations are located at s= $-5\pm j.5$ as shown in Figure 3.4.b. Figure 3.4.c shows the compensated system root loci with dominant roots at s= $-2\pm j2$ utilizing a root locus gain = 13,010. Clearly the roots have not been significantly moved from their natural open loop locations as depicted in Figure 3.4.c. The compensated system roots are located at

$$s= -1.96\pm j2.02,\ -4.92,\ -5.08,\ -9.85,\ -77.7,\ -470 \quad (3.10)$$

The compensated system step response and BODE diagram are shown in Figure 3.4.d and Figure 3.4.e respectively.

C. N-2 FEEDBACK STATES

When N-2 states are available to be measured and fedback, the system designer may choose locations for N-3 roots. The G(s)H(s) function will have three excess poles that will follow asymptotic angles of -180° and ±60°. The roots originating at these three excess poles will attempt to go to infinity as the gain approaches infinity. The designer is now primarily concerned with the roots originating at the two excess poles that will naturally follow asymptotic angles of ±60°. As the gain is increased, these roots will move toward the right half plane, affect the dominance of the dominant roots, or perhaps even cause system instability. Clearly in designing a system that has N-2 states available for

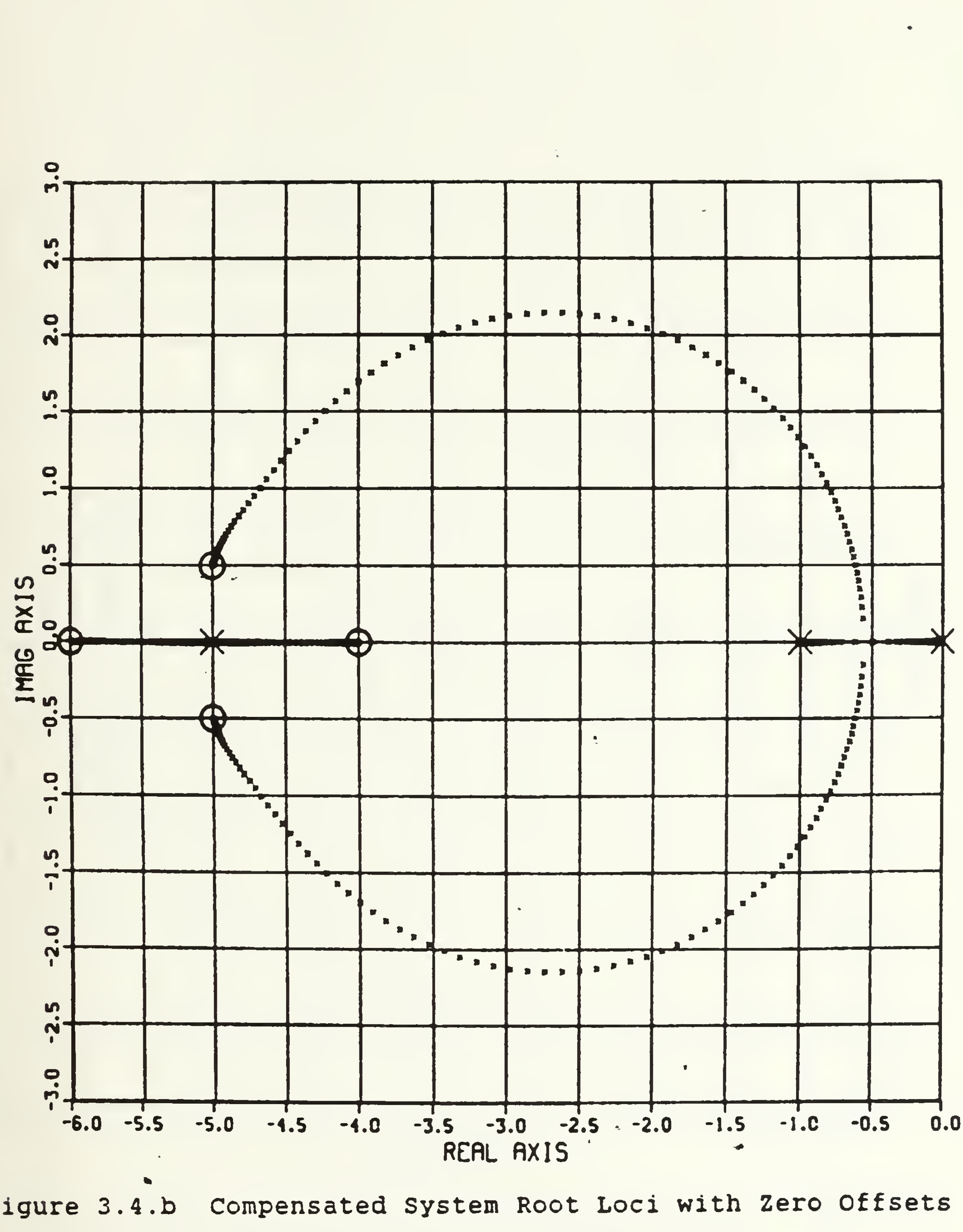

Figure 3.4.b Compensated System Root Loci with Zero Offsets

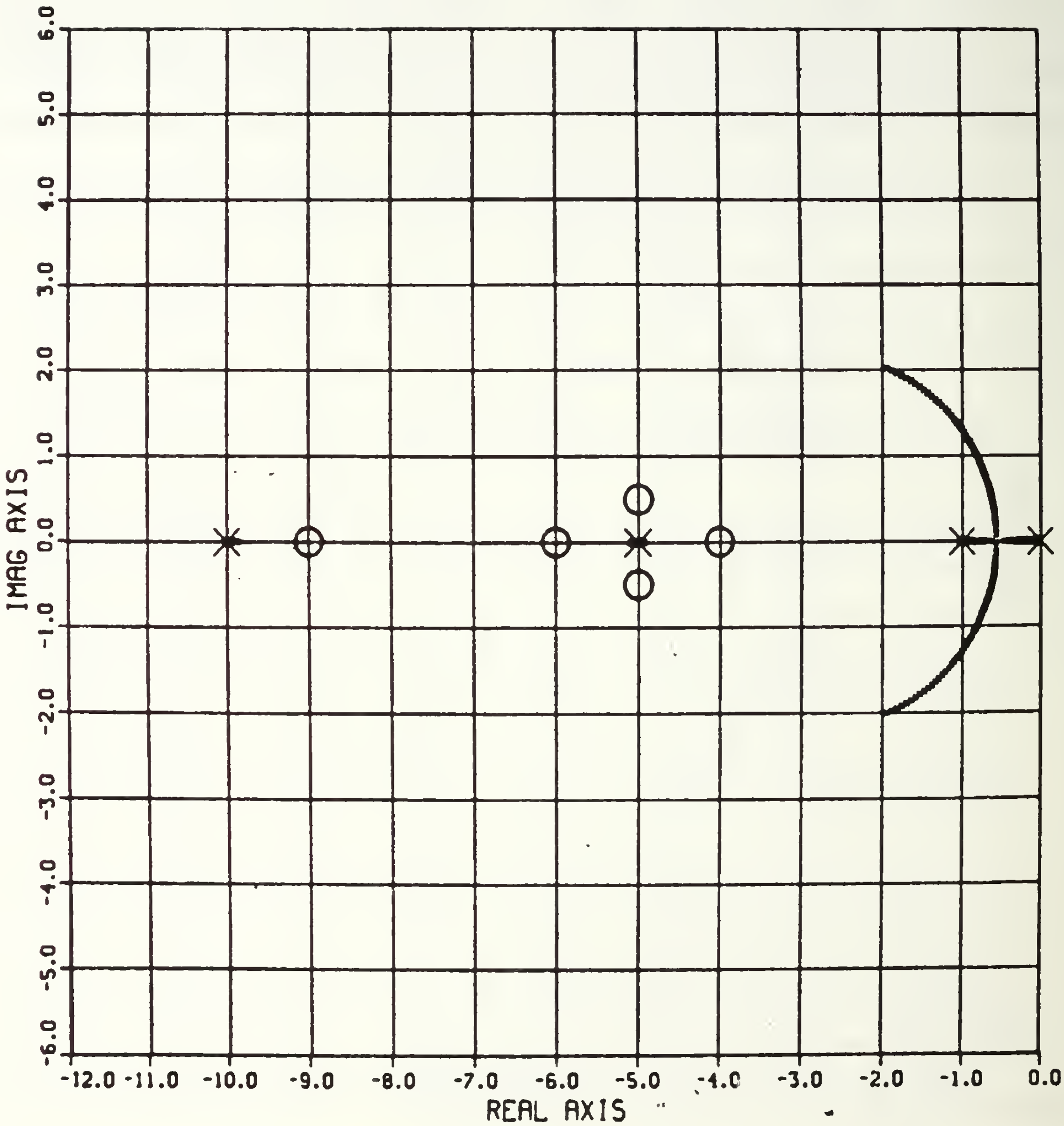

Figure 3.4.c Final Compensated System Root Locus Plot

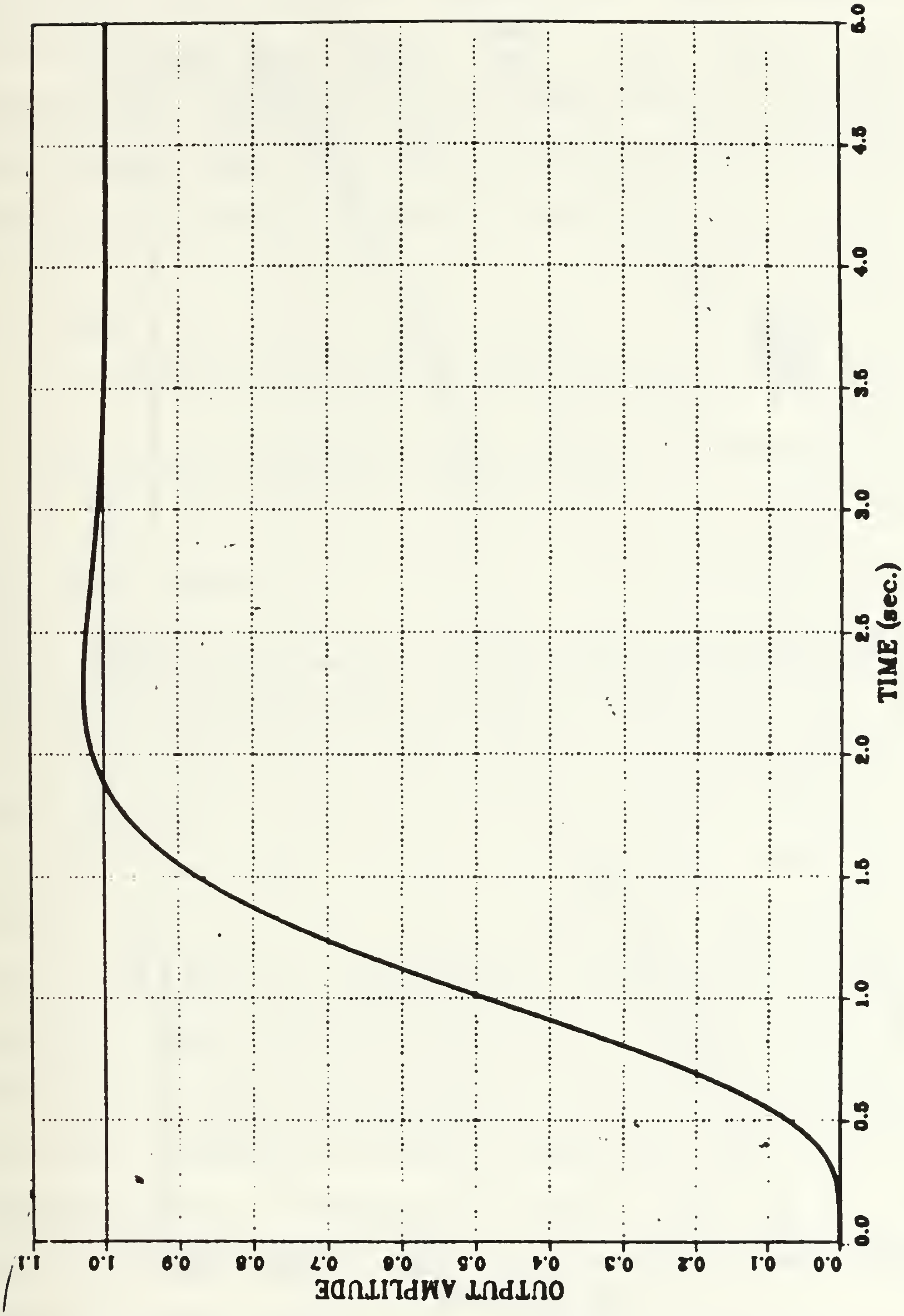

Figure 3.4.d Compensated System Step Response

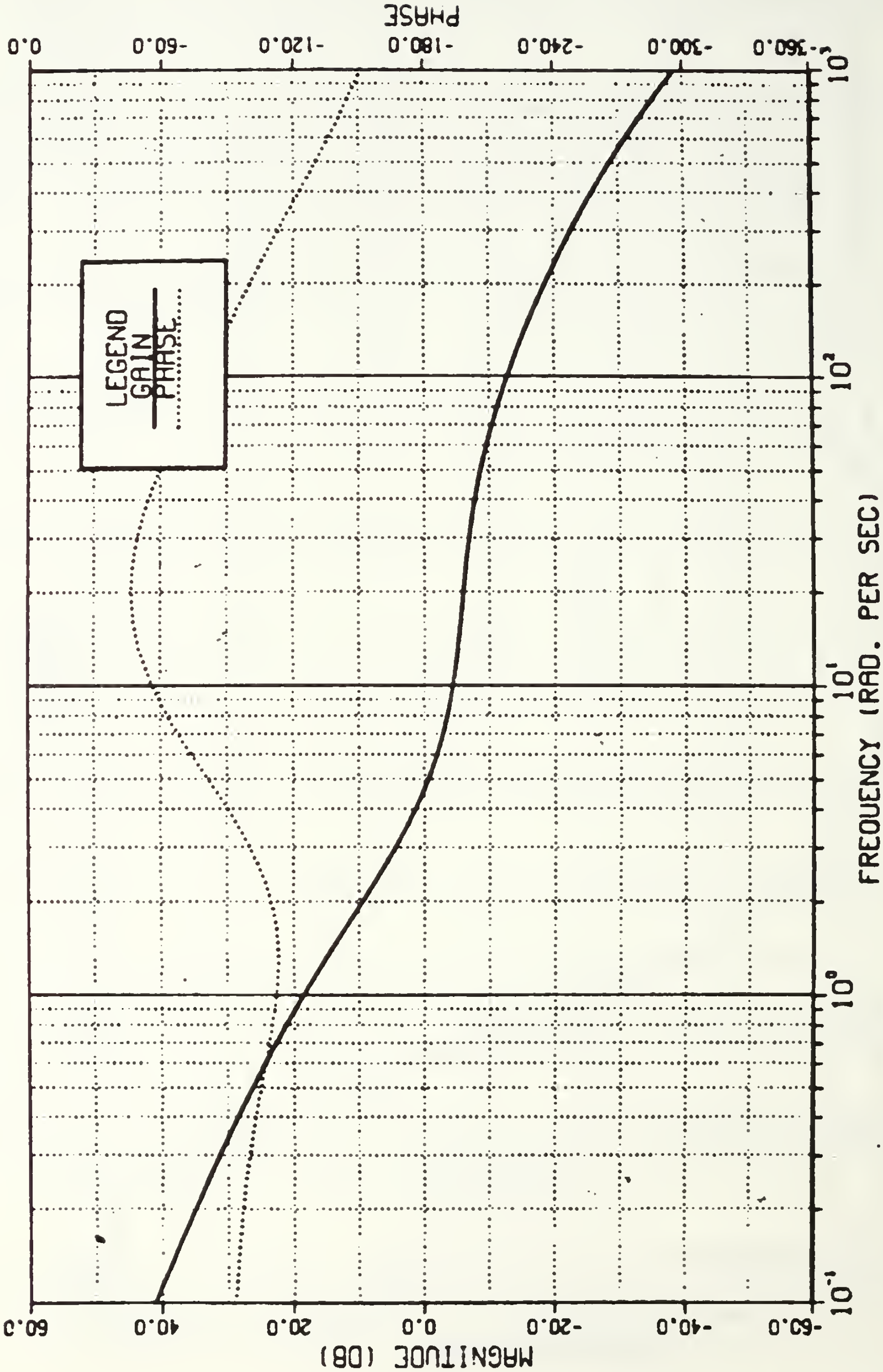

Figure 3.4.e Compensated System BODE Diagram

feedback, the system designer should attempt to control those roots originating at poles nearest the chosen dominant root locations whenever possible.

EXAMPLE 3.5 Equation 3.11 defines the G(s) function for a sixth order plant, and the G(s)H(s) function with dominant roots at s= -2±j2 is defined by equation 3.12.

$$G(s) = \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)} \tag{3.11}$$

$$G(s)H(s) = \frac{K(s+2-j2)(s+2-j2)(s+r_3)}{s(s+1)(s-5)^2(s+10)(s+50)} \tag{3.12}$$

The three unspecified roots will follow asymptotic angles of -180° and ±60° as the gain of the system approaches infinity. The remaining specifiable root is chosen at s= -4. This is not an arbitrary choice. Two of the unspecified roots will have asymptotic angles of ±60° as the system gain is increased. We cannot control the distance that these two unspecified roots will travel toward the right half plane as the system gain is increased. Consequently, the system designer wants these uncontrollable roots to be as far left of the jw axis as possible. This will ensure system stability and hopefully retain a dominant role for the chosen dominant roots. Choosing the third root at s= -4 will guarantee that the two unspecified roots following asymptotic angles of ±60° will initiate from poles at s= -5 and s= -10

from classical root locus evaluation [Ref. 3]. Placing this zero of the G(s)H(s) function (s+4) between any other pole-pole combination in this particular case would result in less than optimum results using root placement with only partial state feedback. The pole at s= -50 will follow an asymptotic angle of -180°. The root locus plot for the initial compensation scheme is shown in Figure 3.5.a. The numerical value for the summation of the three specifiable roots is 8, while a numerical value of 63 remains for the sum of the three unspecified root locations. Figure 3.5.b shows the general shape of the dominant root loci as the zeros of the G(s)H(s) function approach the poles of the G(s)H(s) function as the gain approaches infinity. Acceptable zero offset locations are located at s= -3±j1.2 as shown in Figure 3.5.c. Figure 3.5.d shows the compensated system root locus plot with a root locus gain of 2,512. The compensated system block diagram is shown in Figure 3.5.e. The roots of the compensated system are located at

$$s= -1.96\pm j1.86,\ -4.28,\ -5.79\pm j5.41,\ s= -51.2 \qquad (3.13)$$

The compensated system step response and BODE diagram are shown in Figures 3.5.f and 3.5.g respectively.

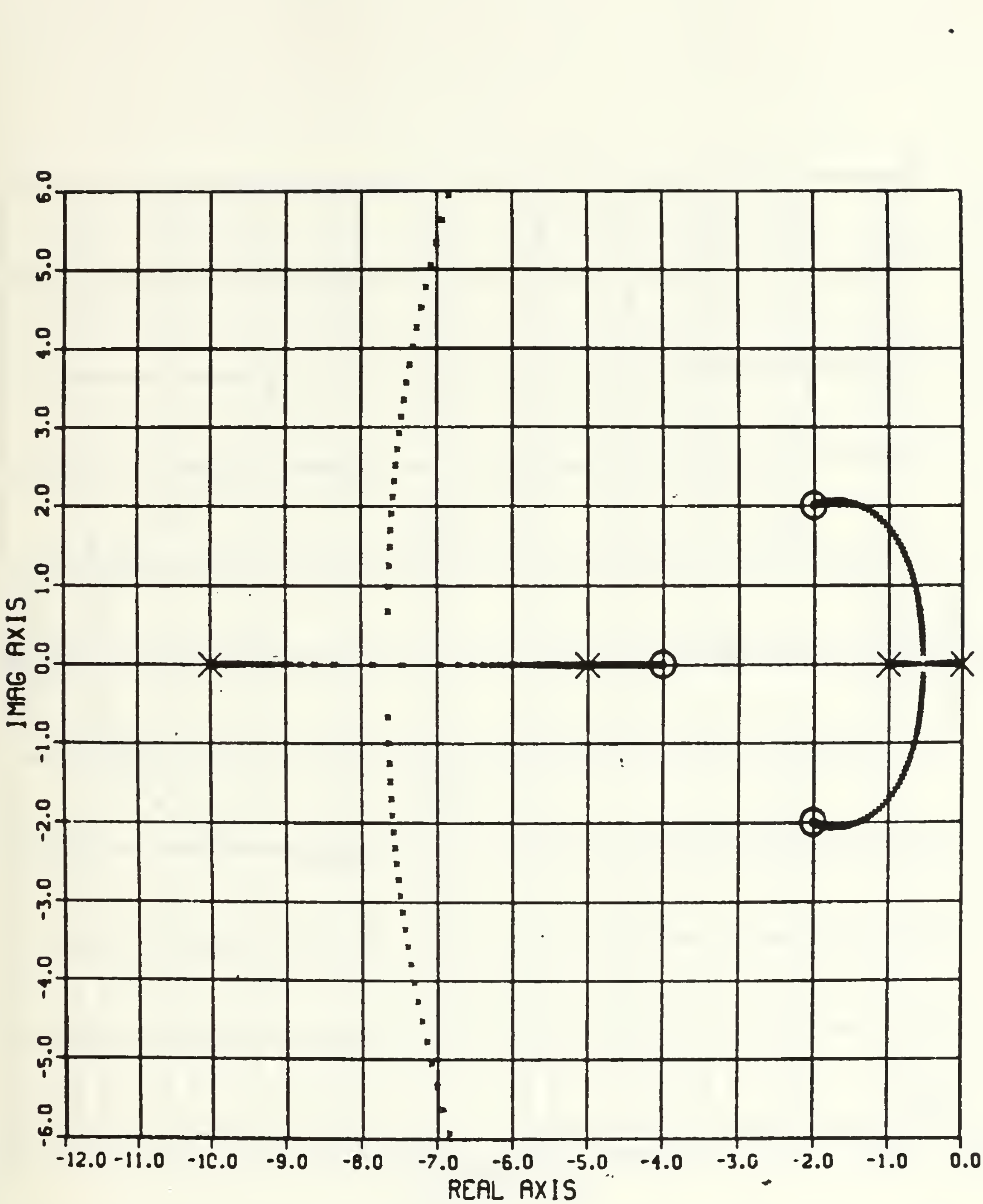

Figure 3.5.a Compensated System Root Loci without Zero Offsets

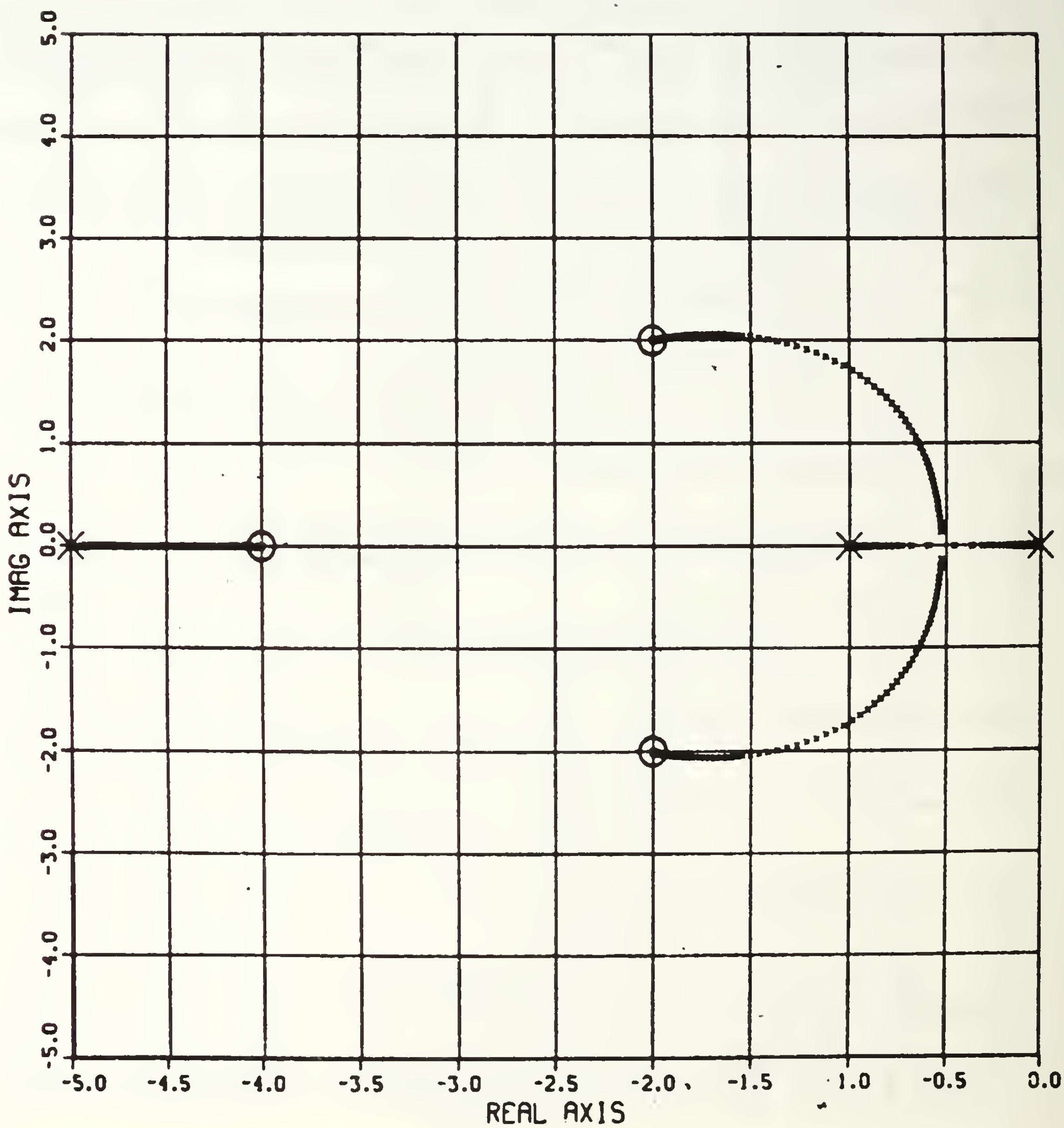

Figure 3.5.b Root Loci for Dominant Roots without Zero Offsets

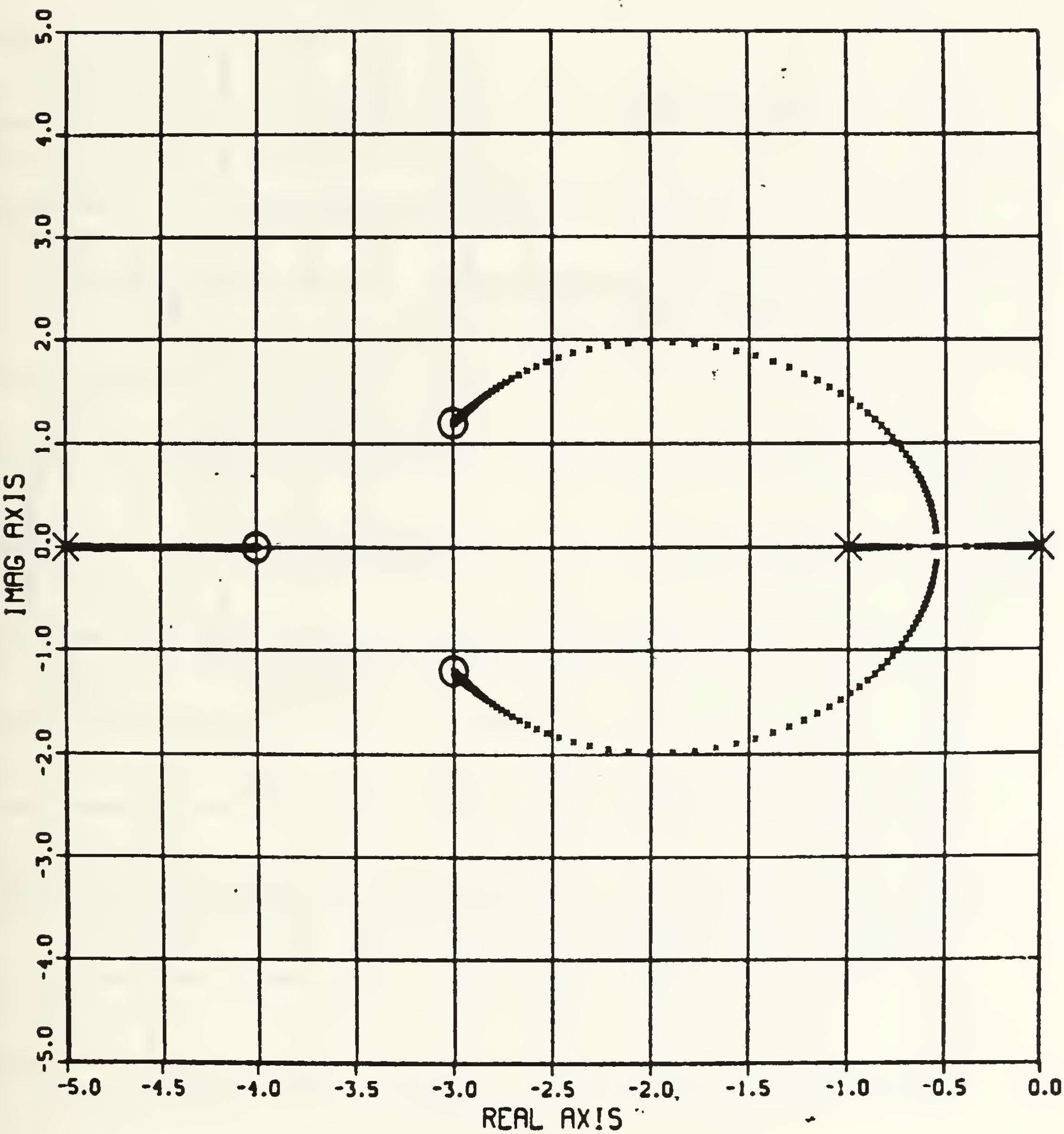

Figure 3.5.c Compensated System Root Loci with Zero Offsets

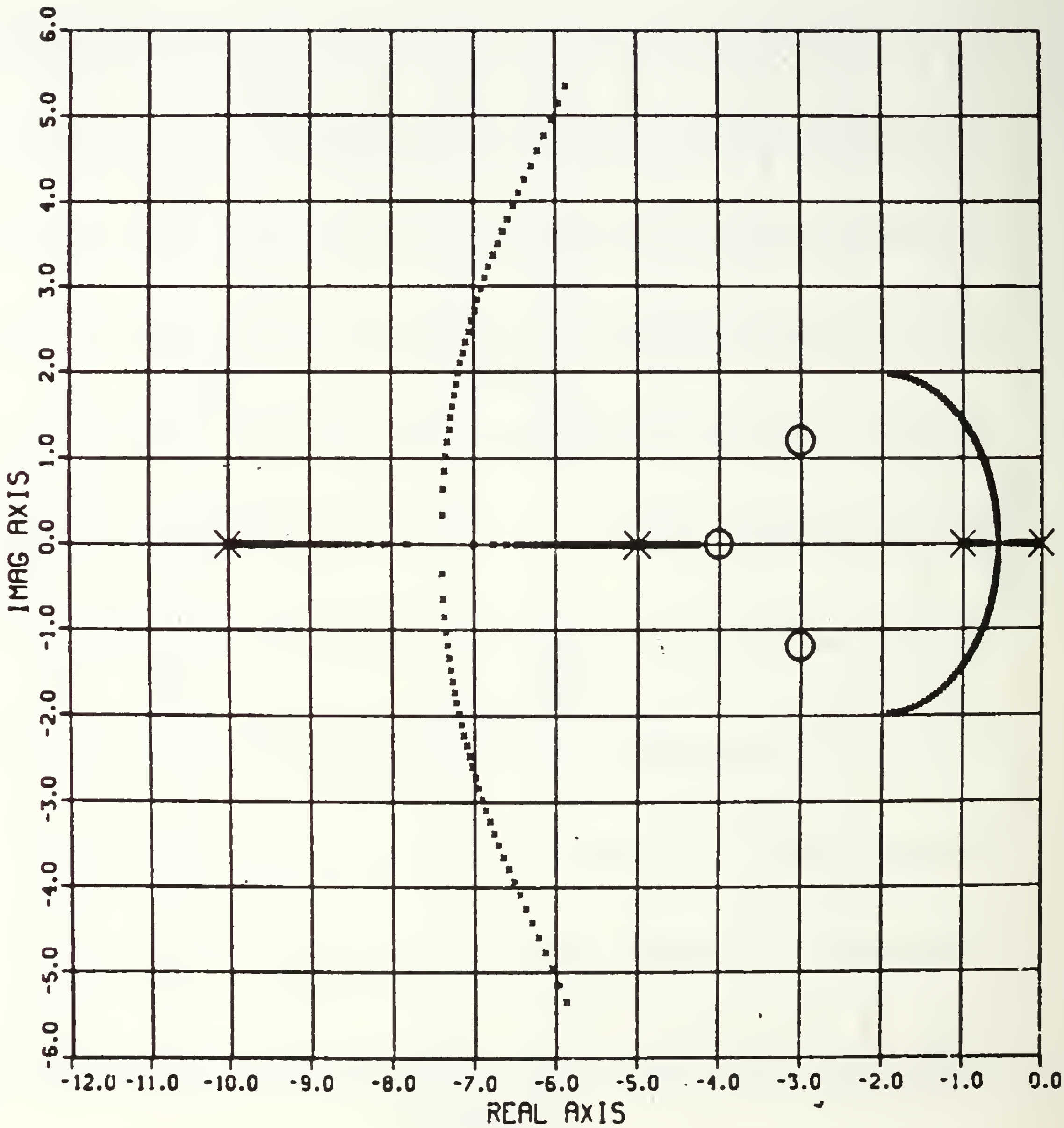

Figure 3.5.d Final Compensated System Root Locus Plot

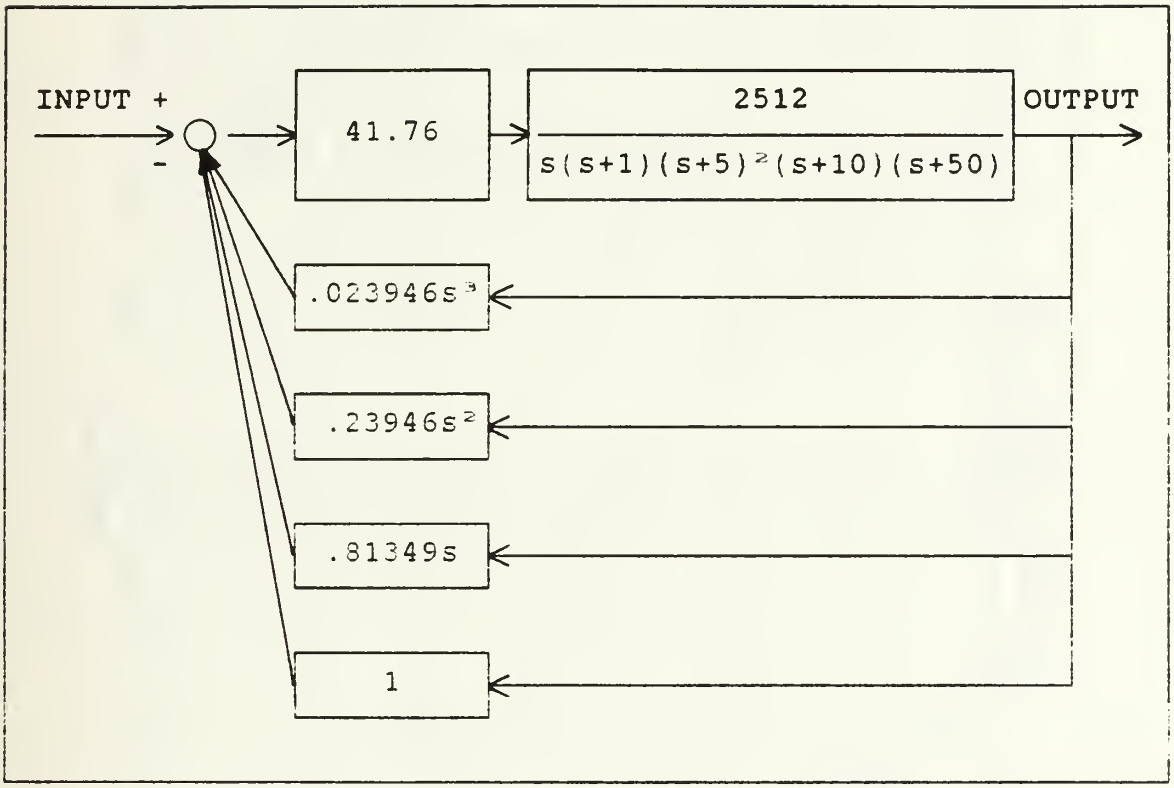

Figure 3.5.e Compensated System Block Diagram

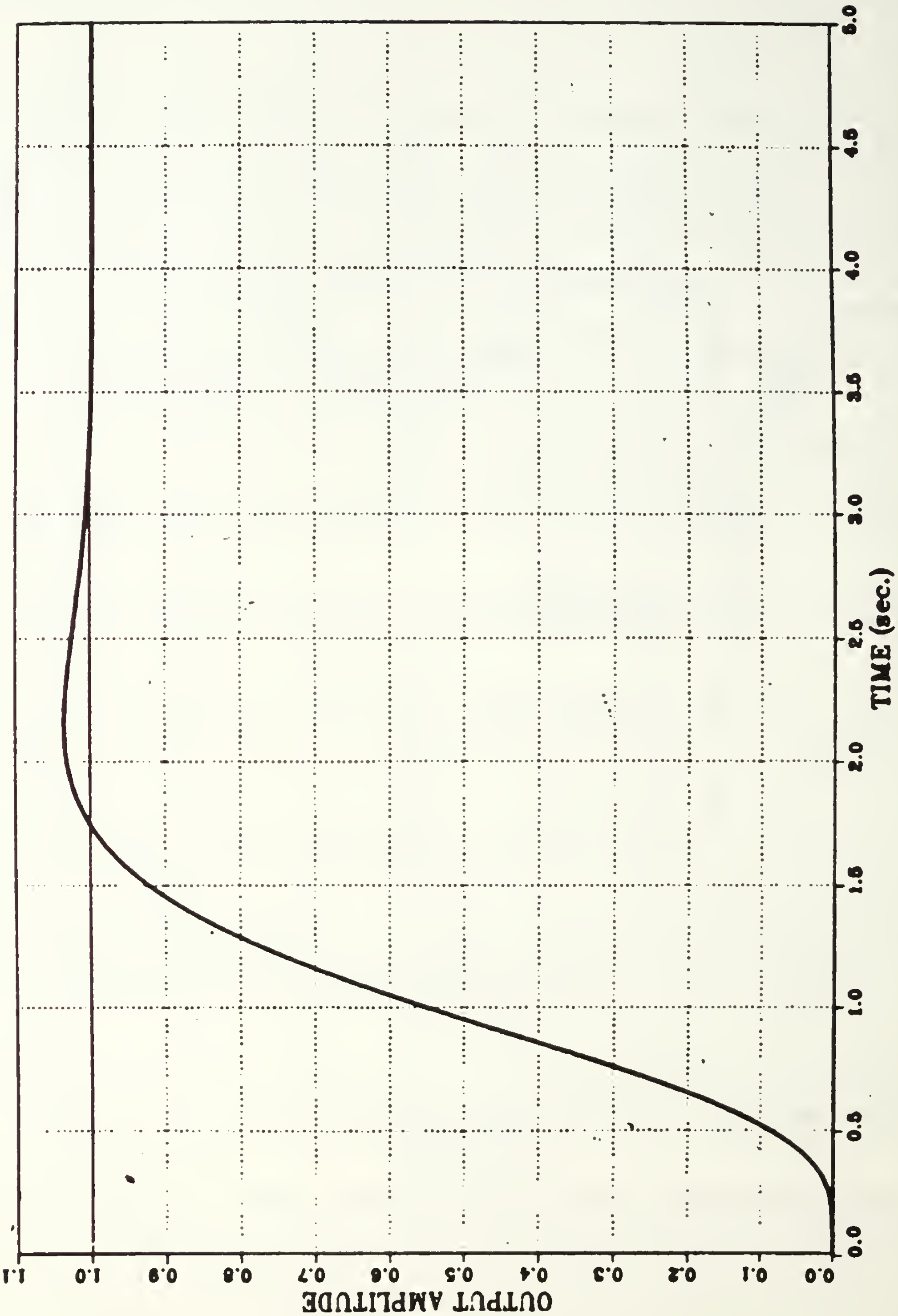

Figure 3.5.f Compensated System Step Response

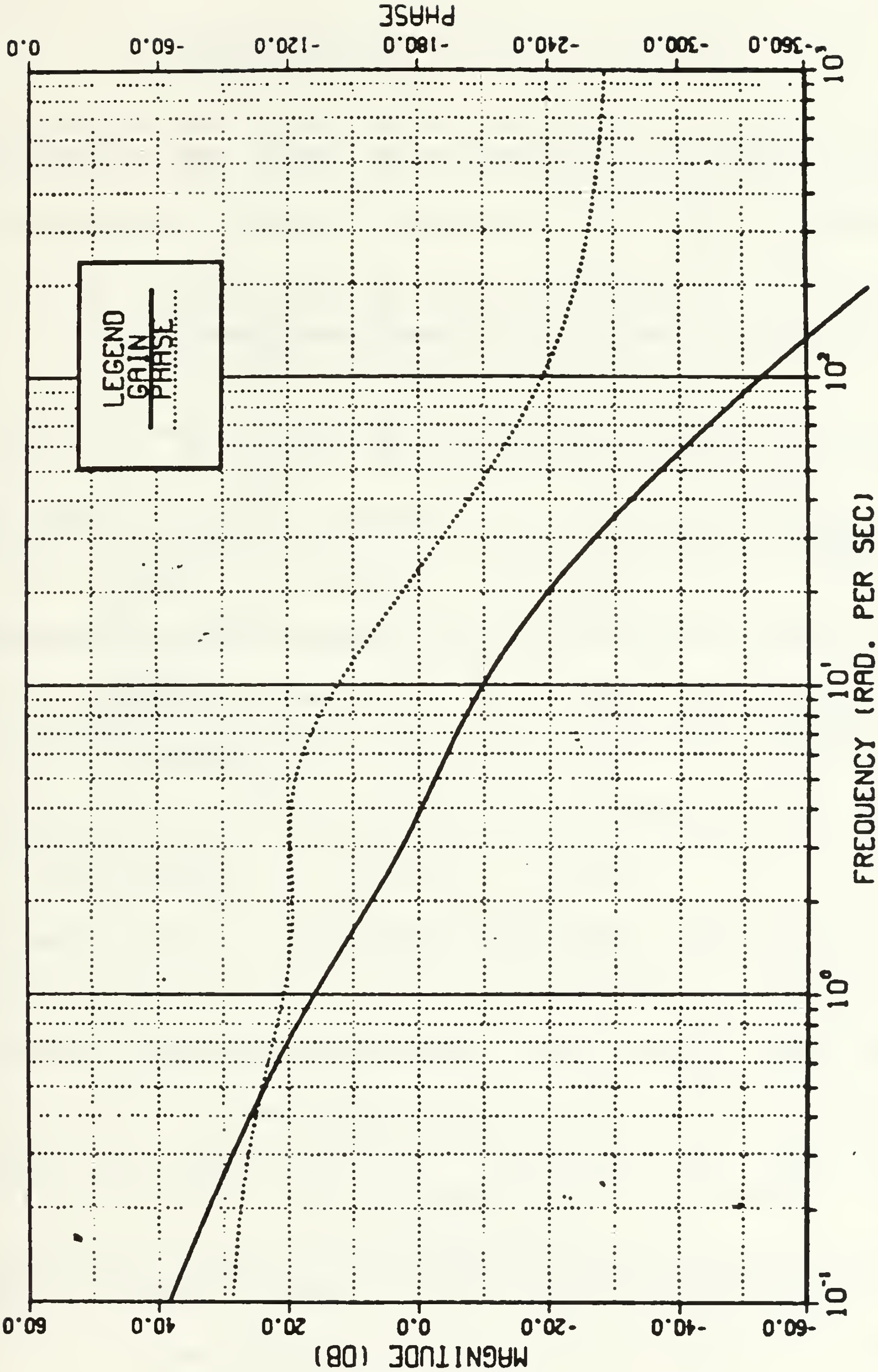

Figure 3.5.g Compensated System BODE Diagram

EXAMPLE 3.6

$$G(s) = \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \qquad (3.14)$$

With dominant roots chosen at s= -2±j2, the G(s)H(s) function becomes

$$G(s)H(s) = \frac{K(s+2+j2)(s+2-j2)(s+r_3)(s+r_4)}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \qquad (3.15)$$

Roots r_3 and r_4 are chosen at s= -4 and s= -6. The sum of the open loop poles is 571, and the sum of the four specified roots is 14. The three unspecified roots will have a numerical sum of 557. From classical root locus evaluation it is clear that the roots following asymptotic angles of ±60° will break away from the real axis between s= -10 and s= -50. Zero offset locations at s= -4.3±j.4 are shown in Figure 3.6.a. Figure 3.6.b shows the compensated root locus plot as the gain approaches infinity. A root locus gain of 144,544 places the dominant roots at s= -2±j2 as shown in the final root locus plot depicted in Figure 3.6.c. The compensated system has roots located at

$$s= -2.04\pm j2.09,\ -4.82,\ -5.26,\ -14.43,\ -41.82,\ -501.1 \qquad (3.16)$$

The compensated system step response and BODE diagram are shown in Figures 3.6.d and 3.6.e respectively.

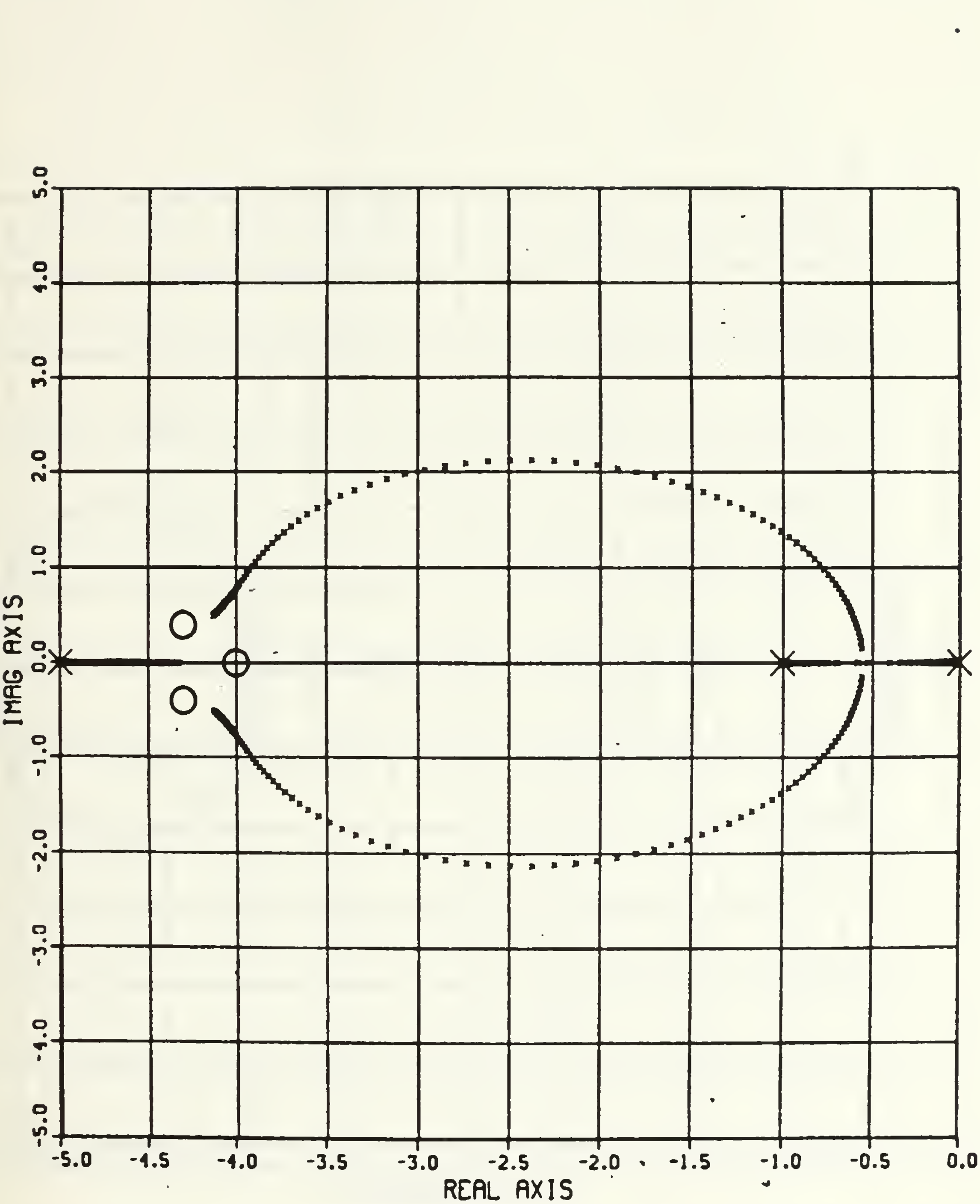

Figure 3.6.a Root Loci for Dominant Roots with Zero Offsets

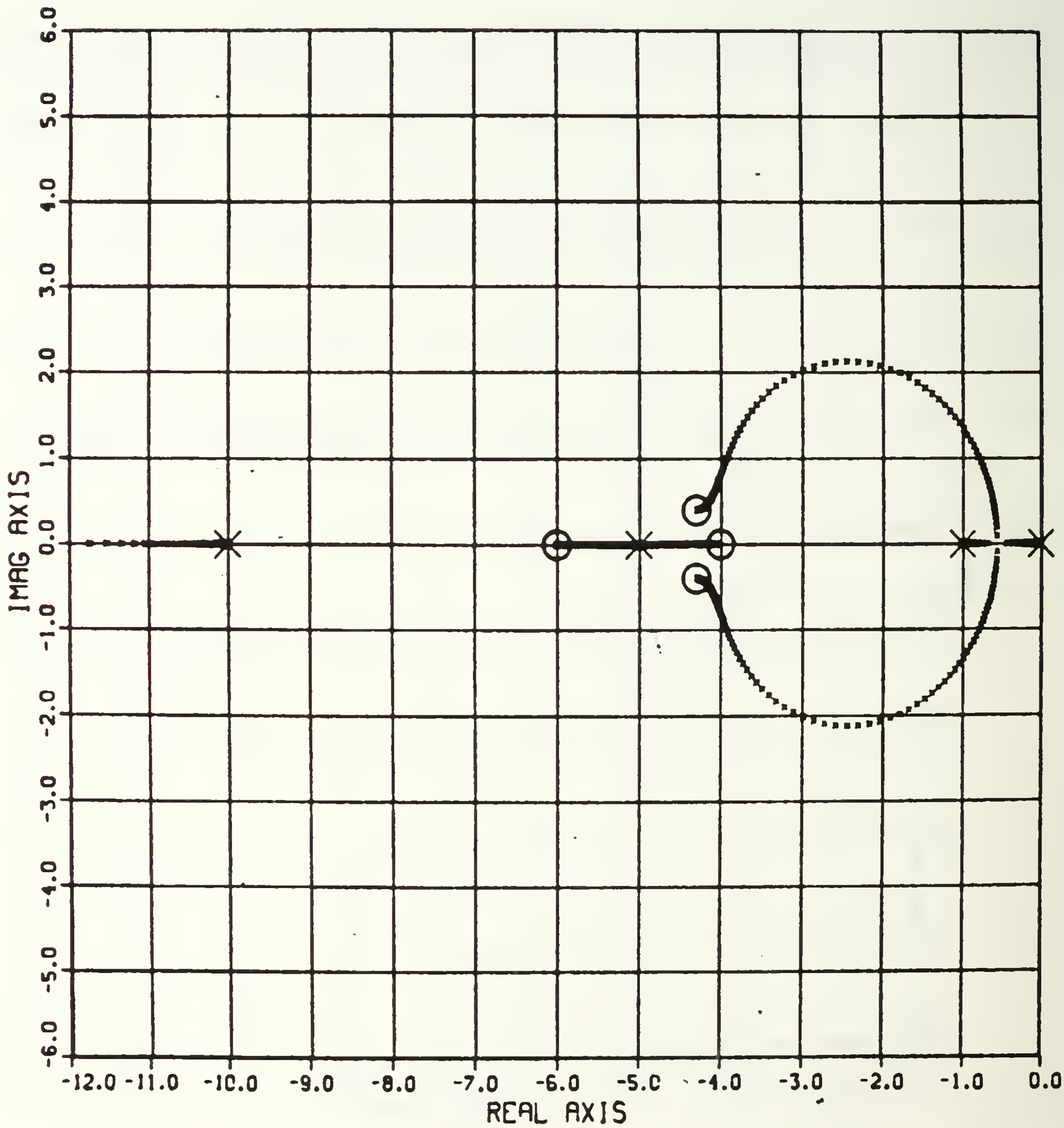

Figure 3.6.b Compensated System Root Loci with Zero Offsets

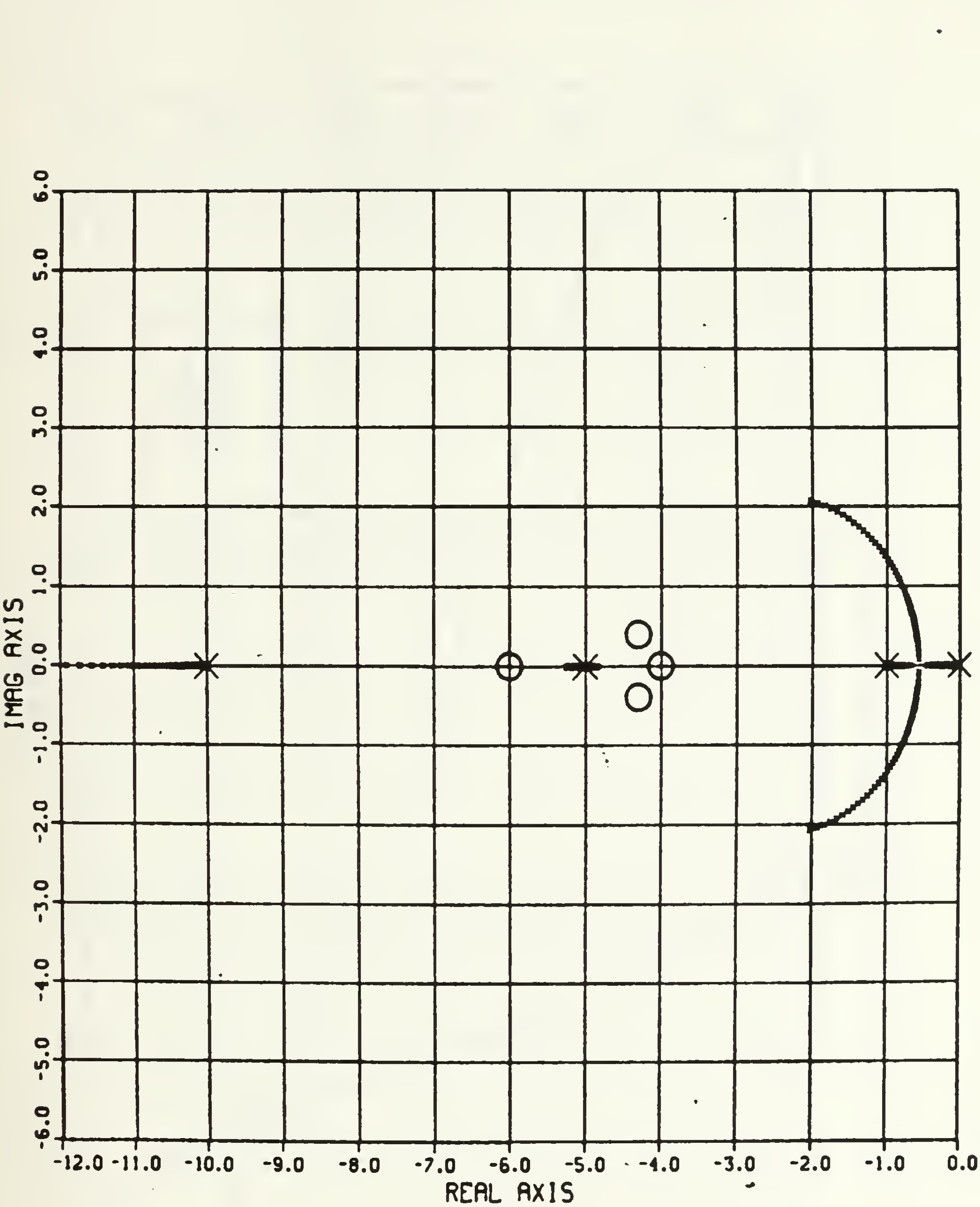

Figure 3.6.c Final Compensated System Root Locus Plot

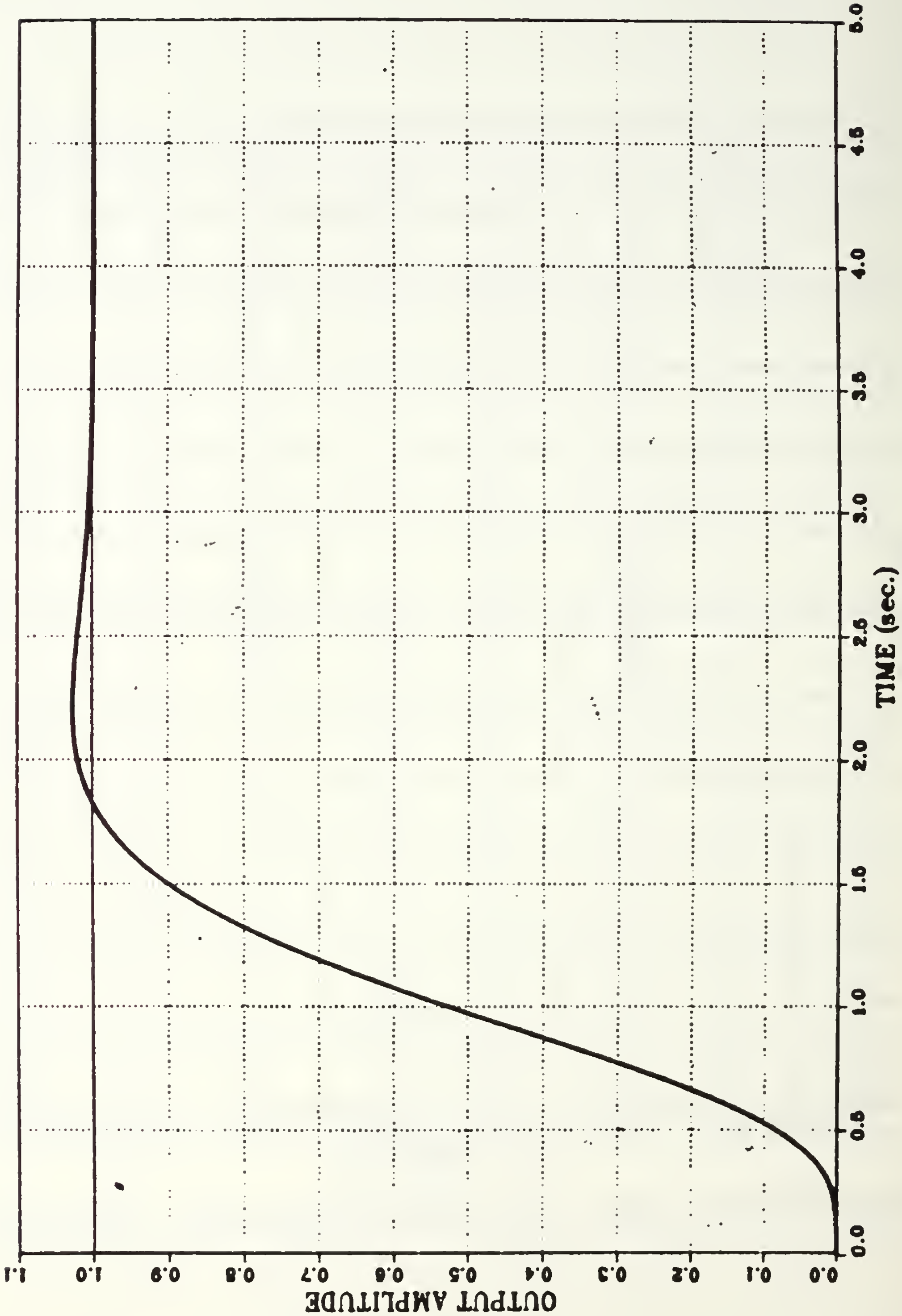

Figure 3.6.d Compensated System Step Response

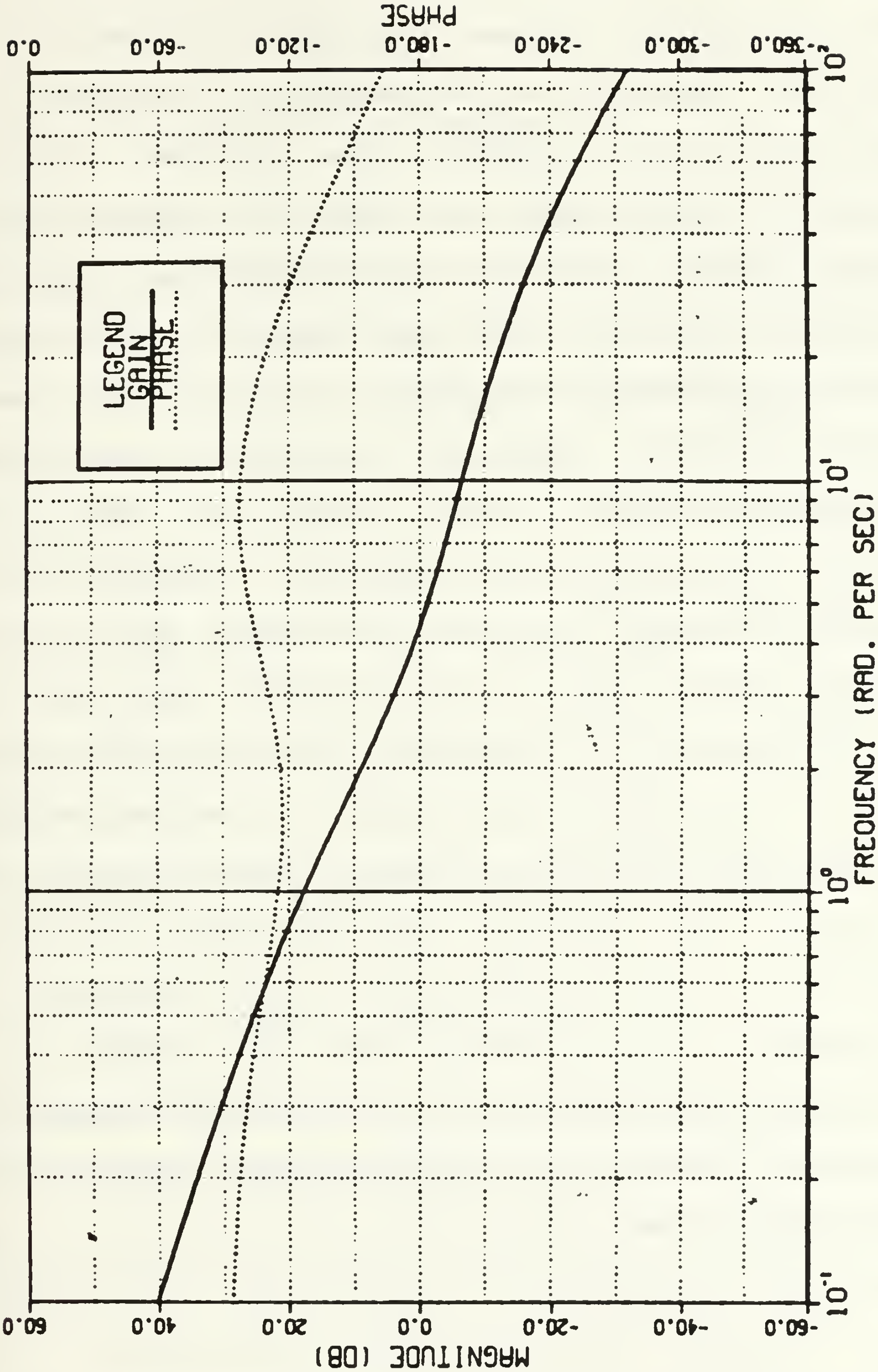

Figure 3.6.e Compensated System BODE Diagram

D. N-3 OR FEWER FEEDBACK STATES

With N-3 or fewer states available to be measured and fedback, the system designer may choose locations for N-4 or fewer roots depending on the particular problem. With N-3 states available for feedback, the G(s)H(s) function will have four excess poles and the roots originating at these poles will follow asymptotic angles of ±45° and ±135° towards infinity as the gain approaches infinity. The unspecified roots following asymptotic angles of ±135° will not cause stability problems. However, the roots following asymptotic angles of ±45° are of crucial concern in terms of system stability and dominant root locations. Clearly as the number of states available to feedback decreases, roots originating at excess pole locations move toward the right half plane at more acute angles[7]. Consequently, as the number of states available to feedback decreases, it becomes increasingly difficult to ensure system stability, and even more difficult to retain the dominance of the chosen dominant root locations. In those cases where system compensation using <u>only</u> partial state feedback does not satisfy the time performance requirements of the system, the designer should consider a combination of partial state feedback and other compensation schemes.

---

[7]See Appendix B

EXAMPLE 3.7 The $G(s)$ function for a sixth order plant is defined in equation 3.17.

$$G(s)=\frac{K}{s(s+1)(s+5)^2(s+10)(s+50)} \tag{3.17}$$

With N-3 states available to feedback, the designer may choose only the dominant root locations. Once again it is assumed that the dominant root locations are required to be located at $s=-2\pm j2$ to meet given system time performance specifications. The four unspecified roots will attempt to go to infinity at asymptotic angles of ±135° and ±45° as the gain is increased. The uncompensated system root locus plot is shown in Figure 3.7.a. Zero offset locations at $s=-1.3\pm j1.1$ are shown in Figure 3.7.b, and the compensated system root locus plot with a root locus gain of 12,022 is shown in Figure 3.7.c. Note that the roots at $s=-2\pm j2$ no longer maintain their dominant system role. The roots of the compensated system are located at

$$s=-2\pm j2,\ -1.8\pm j3.4,\ -49.9,\ -13.5 \tag{3.18}$$

The compensated system step response is shown in Figure 3.7.d. If the characteristics of the resultant step response shown in Figure 3.7.d do not satisfy the time performance requirements of the system, the designer should consider using a combination of partial state feedback and other compensation schemes.

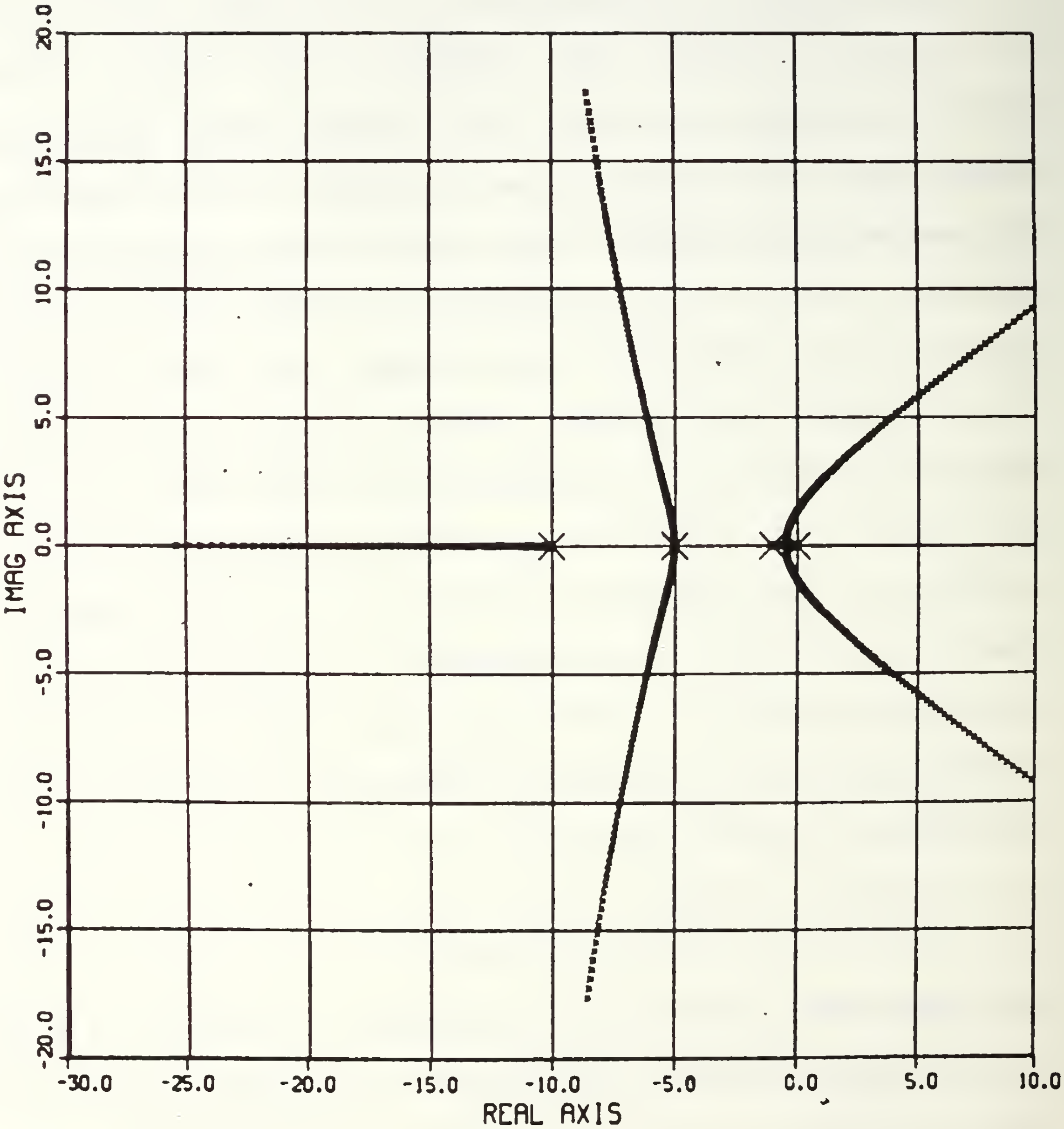

Figure 3.7.a Uncompensated System Root Locus Plot

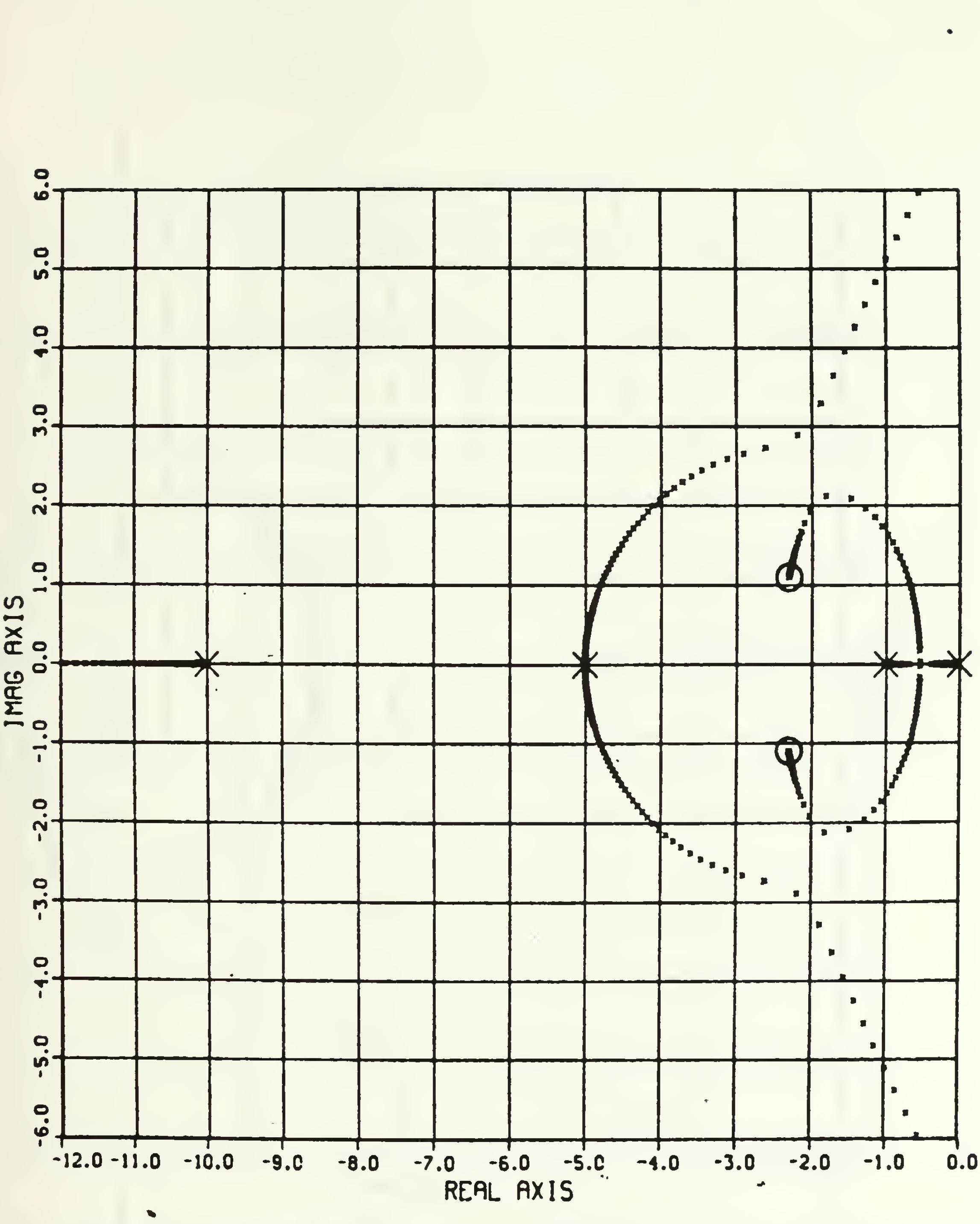

Figure 3.7.b Compensated System Root Loci with Zero Offsets

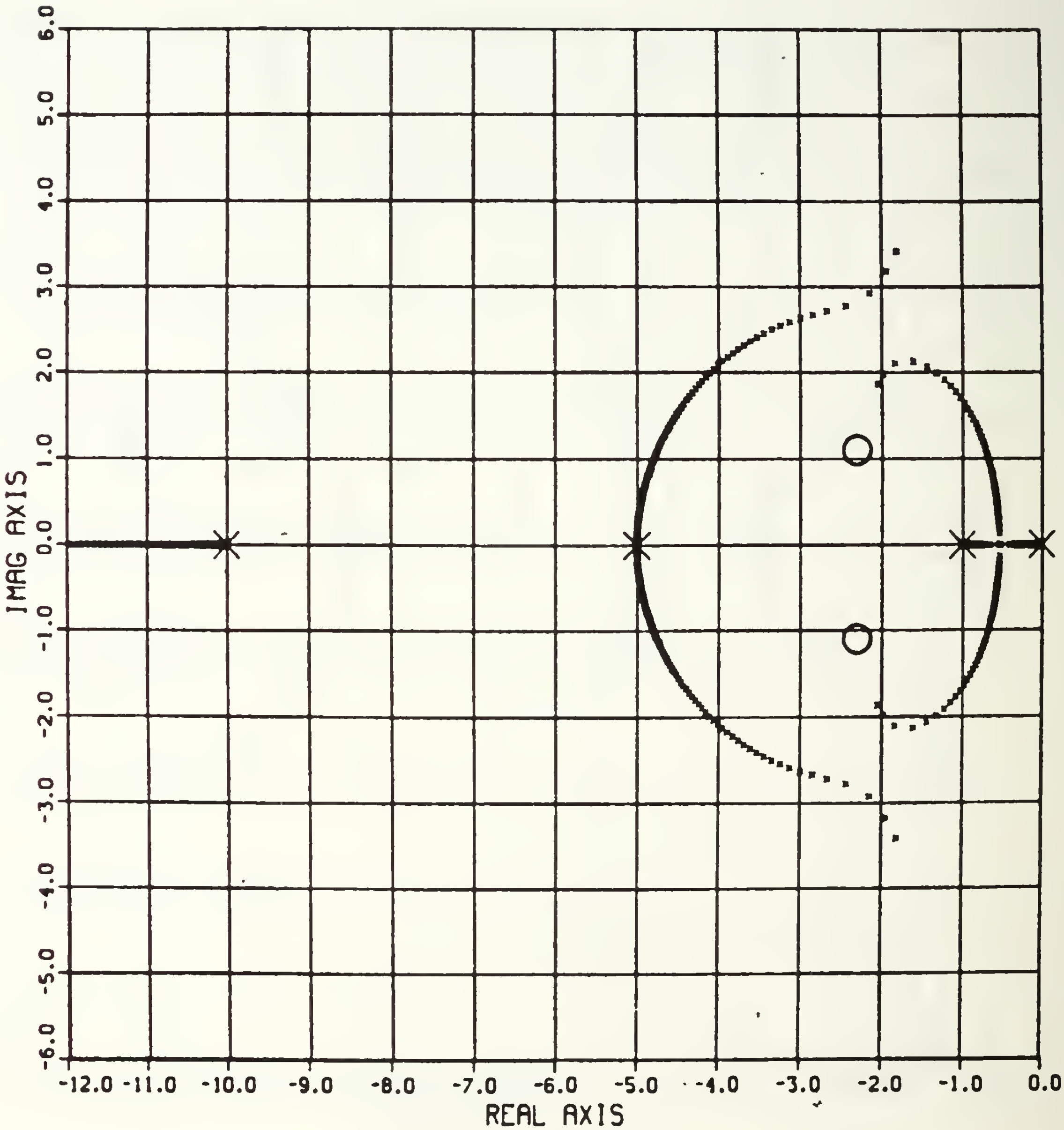

Figure 3.7.c Final Compensated System Root Locus Plot

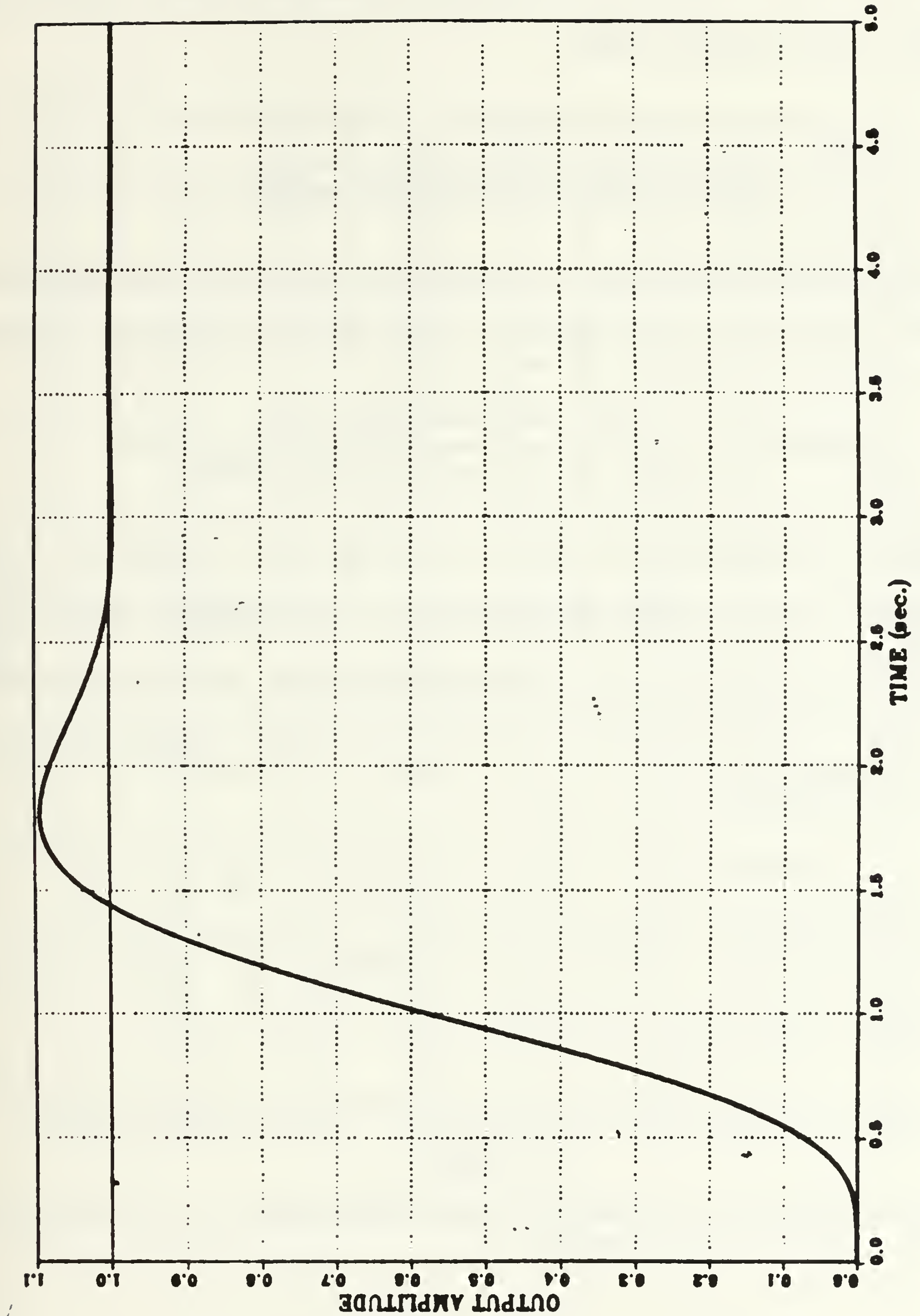

Figure 3.7.d Compensated System Step Response

EXAMPLE 3.8 For the seventh order system in equation 3.19, there are N-3 states available to feedback, and the designer may choose three roots.

$$G(s)= \frac{K}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \tag{3.19}$$

The dominant roots are chosen to meet the time performance requirements of the system. The G(s)H(s) function becomes

$$G(s)H(s)= \frac{K(s+2+j2)(s+2-j2)(s+4)}{s(s+1)(s+5)^2(s+10)(s+50)(s+500)} \qquad (3.20$$

and the compensated system root locus plot is shown in Figure 3.8.a. Zero offset locations at $s= -3.3\pm j.5$ are shown in Figure 3.8.b, and the compensated system root locus plot is shown in Figure 3.8.c. The roots of the compensated system are located at

$$s= -1.96\pm j1.97,\ -4.22,\ -5.80\pm j4.19,\ -51.1,\ -500 \tag{3.21}$$

The compensated system step response is shown in Figure 3.8.d.

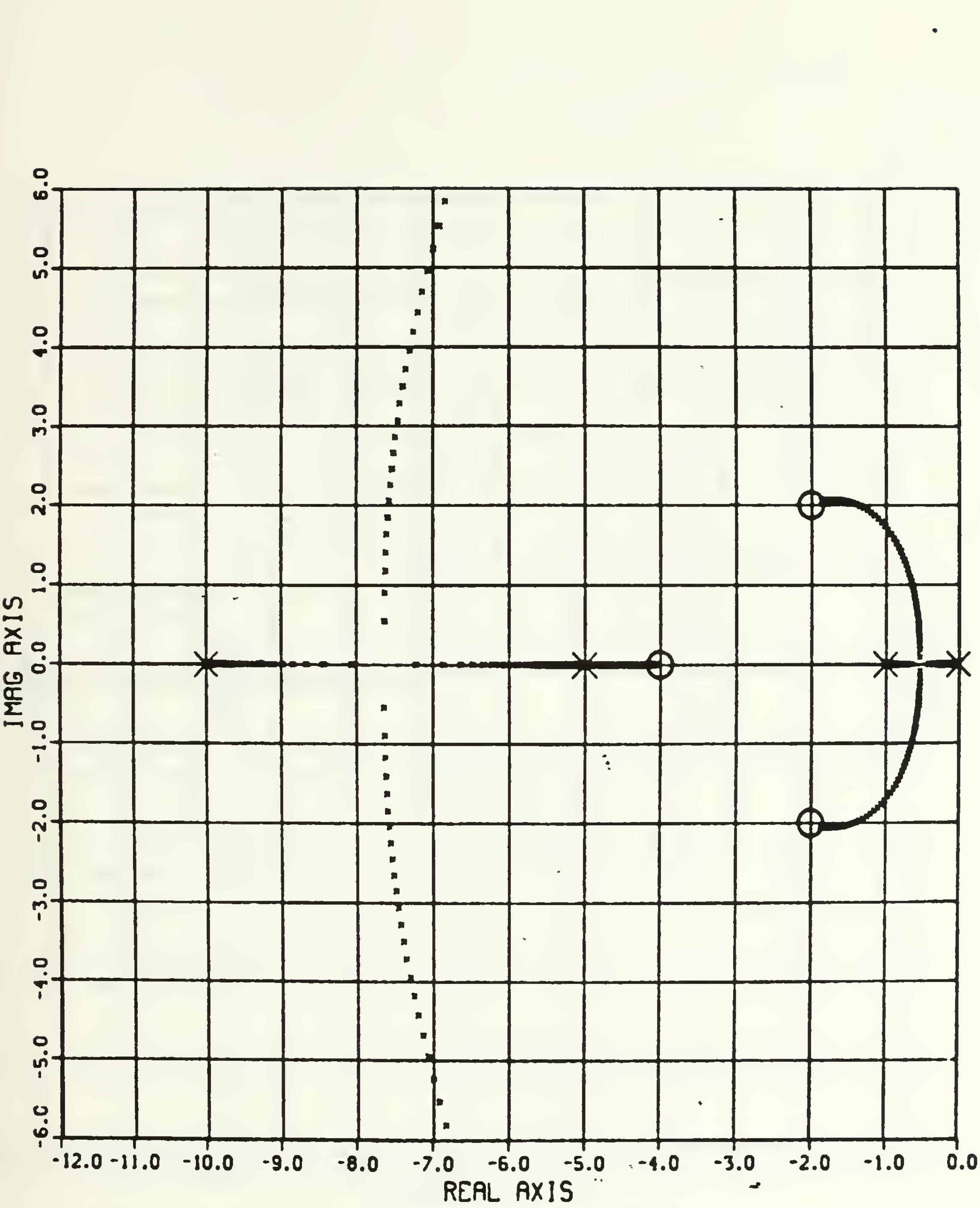

Figure 3.8.a Compensated System Root Loci without Zero Offsets

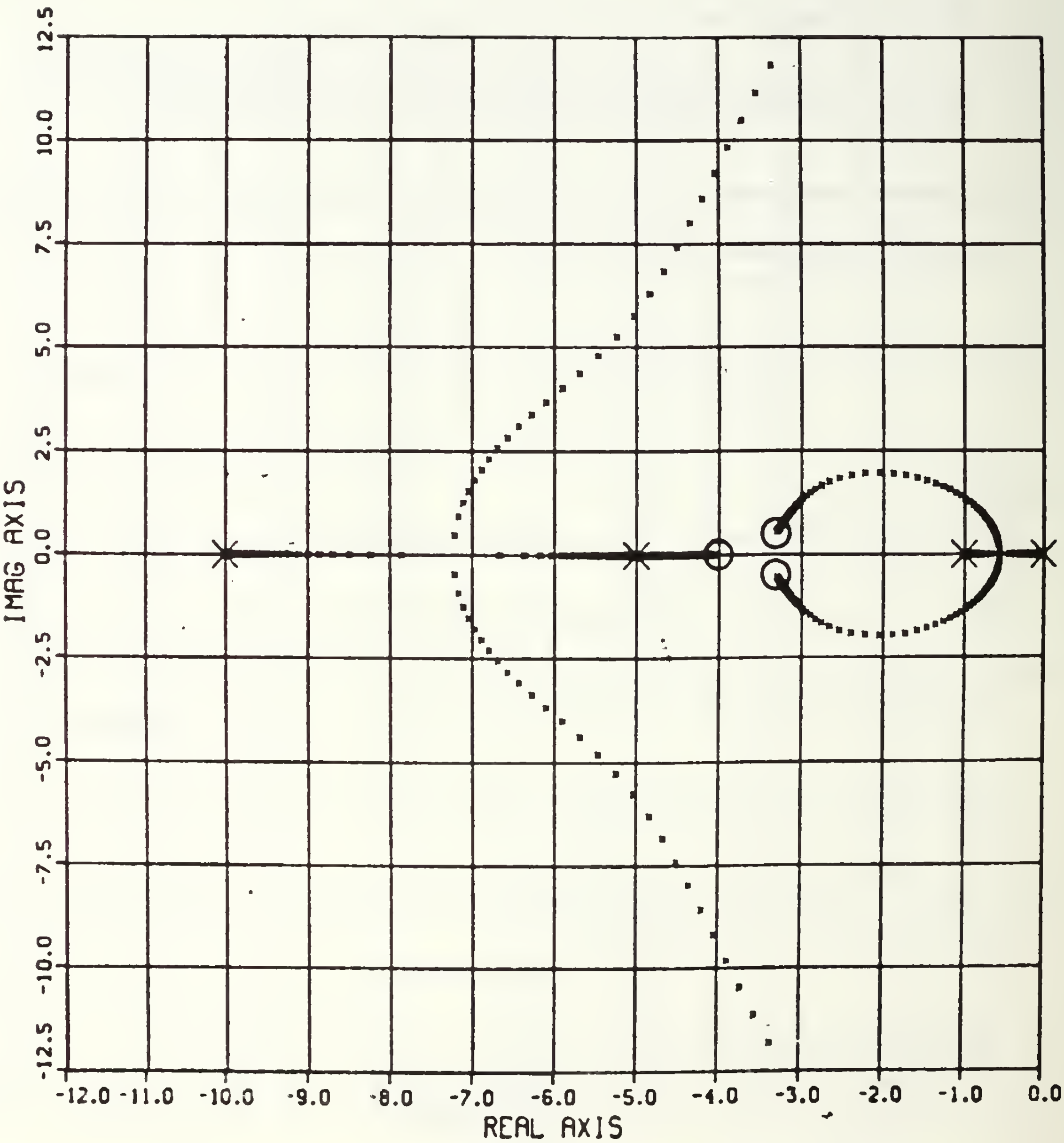

Figure 3.8.b Compensated System Root Loci with Zero Offsets

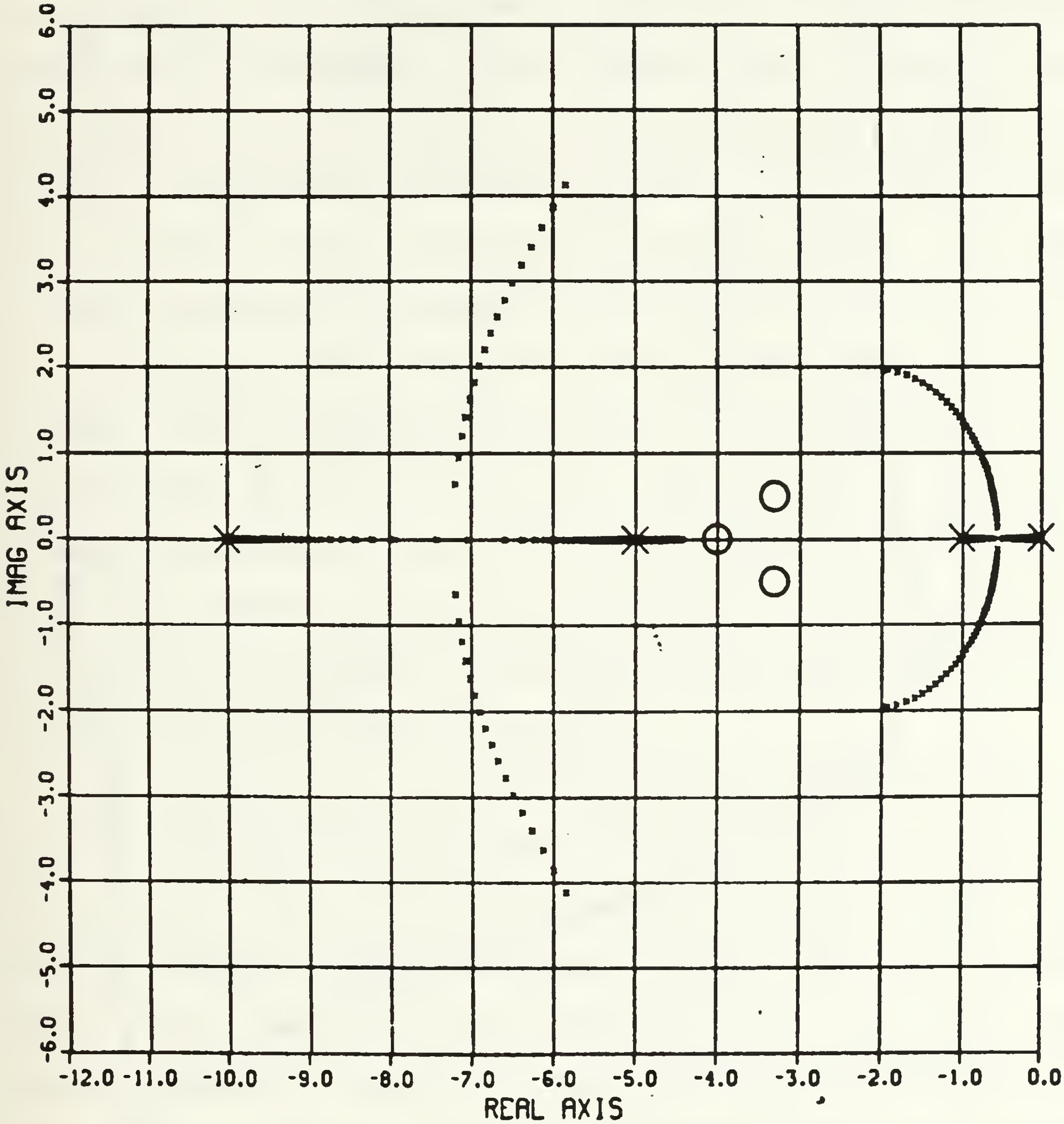

Figure 3.8.c Final Compensated System Root Locus Plot

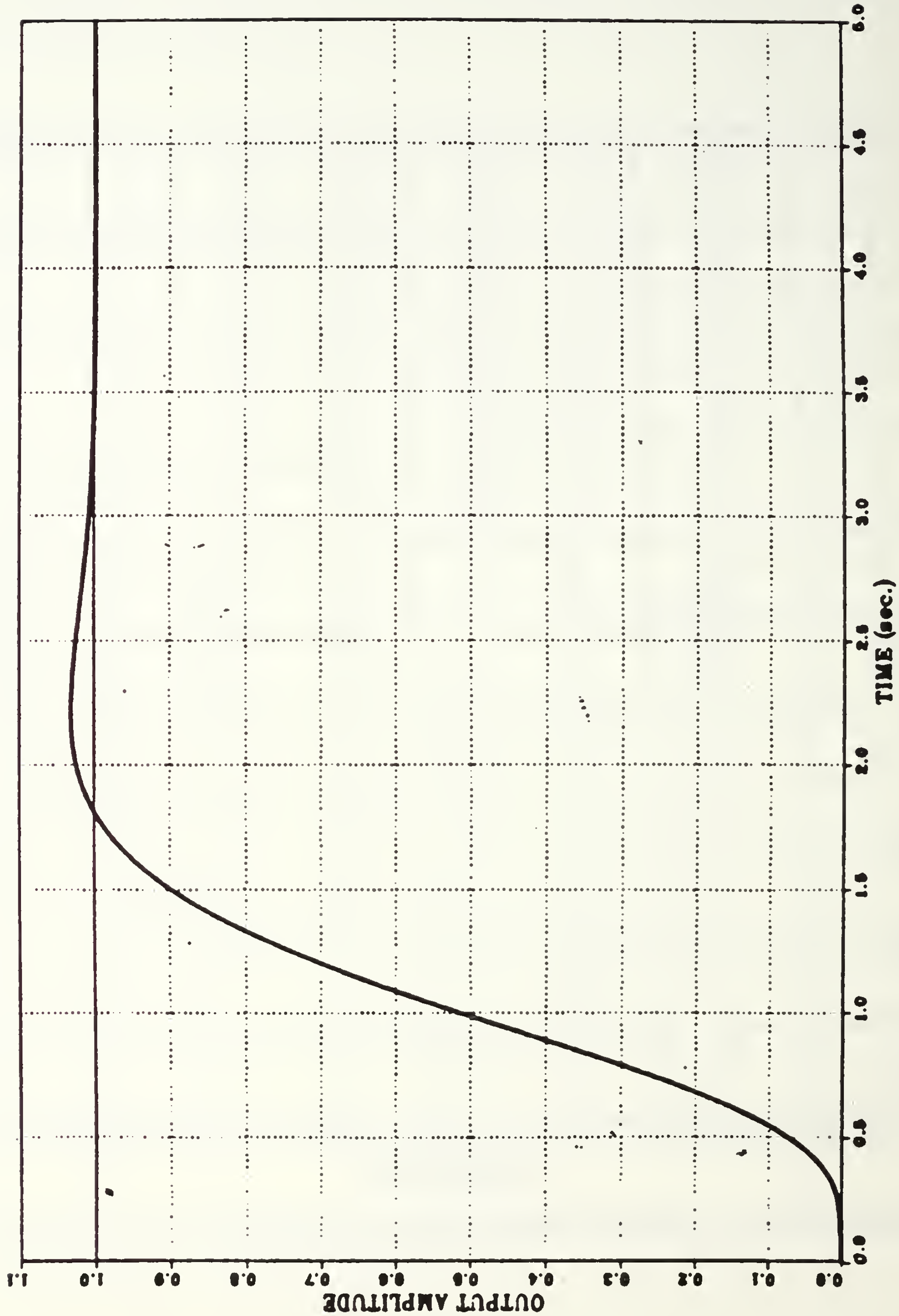

Figure 3.8.d Compensated System Step Response

E. CONCLUSIONS

A procedure for root placement with partial state feedback using transfer function methods was developed in this chapter. It was assumed that only the higher ordered states were not available to be measured and fedback. The coefficient of the N-1 term in the characteristic equation is the sum of the system's open loop poles, and is also the sum of the system roots. With partial state feedback, this sum is fixed and cannot be adjusted. The system's dominant roots are chosen to meet the time performance specifications of the system. The remaining specifiable roots chosen by the designer are chosen such that the roots move as little as possible from their natural open loop pole locations, yet allow the dominant roots to retain their dominant system role. Specific examples are presented throughout the chapter to demonstrate the design procedure.

When N-1 states are available to be measured and fedback, N-2 root locations may be chosen. The system will have two excess poles that will follow asymptotic angles of ±90°. If the two excess poles breakaway from the real axis significantly left of the dominant root locations, the designer should not experience much difficulty satisfying the system specifications. With N-2 feedback states, the system will have three excess poles that will follow asymptotic angles of -180° and ±60°. As the system gain is increased, the roots following asymptotic angles of ±60° will move

toward the right half plane, affect the dominance of the dominant roots, or perhaps cause system instability. Therefore, when N-2 states are available for feedback, the designer should attempt to control those roots originating at poles nearest the chosen dominant root locations. When N-3 or fewer feedback states are available, it becomes more difficult to "control" those roots originating at the system's excess poles. As the number of feedback states decreases, the number of excess poles increases, and the roots originating at these excess poles move toward the right half plane at more acute angles. Consequently it becomes more difficult to ensure that the dominant roots retain their dominant role.

The partial state feedback procedures presented in this chapter using transfer function methods are very applicable techniques. However, the pole-zero composition of the plant will determine the effectiveness of these design procedures. There are plants that cannot be compensated to satisfy given system specifications using only the partial state feedback procedures presented in this chapter. A combination of partial state feedback and other compensation schemes should be considered by the engineer in these cases.

IV. PARTIAL STATE FEEDBACK: PLANT WITH A ZERO

A. INTRODUCTION

In Chapter III a partial state feedback design procedure was developed for root placement using transfer functions for the all pole plant. With partial state feedback, less than all of the system states are available to be fedback. It was shown that as the number of available feedback states decreases, it became increasingly difficult to meet required system specifications using only partial state feedback procedures. If the system plant under study contains a zero, the flexibility of the designer in satisfying the given specifications is enhanced. There are few plants that contain a zero, and those that do contain a zero are built for very specific purposes. However, the same results are obtained by compensating the system with a zero in cascade with the forward path of the system. The partial state feedback procedures developed in the previous chapter apply to the plant that contains a zero, or the system that contains a zero as a result of cascade compensation.

B. APPLIED PROCEDURE

When N-1 states are available to be measured and fedback, the system designer may choose locations for N-2 roots. If the forward path of the system contains a zero, either by design or by cascade compensation, the G(s)H(s) function will

have only one excess pole that will follow as asymptotic angle of -180° as the gain is increased. Consequently the design problem in this case reduces to one that utilizes the full state feedback procedures discussed in Chapter II!

EXAMPLE 4.1 The fourth order system designed in example 2.1 now contains a zero as defined by equation 4.1.

$$G(s)= \frac{K(s+9)}{s(s+5)(s+5)(s+10)} \tag{4.1}$$

With only N-1 feedback states, the system designer may choose only the dominant root locations. Dominant root locations are again chosen arbitrarily at s= 2±j2 to satisfy arbitrary time performance and bandwidth requirements of the system. The resultant G(s)H(s) function is defined by equation 4.2.

$$G(s)H(s)= \frac{K(s+2+j2)(s+2-j2)(s+9)}{s(s+5)(s+5)(s+10)} \tag{4.2}$$

Equation 4.2 is exactly the same as equation 2.6. The partial state feedback design problem is reduced to one that follows the full state feedback procedures discussed in Chapter II.

The designer has an additional chosen root when designing a system with partial state feedback if the forward path of the system contains a zero. Note that the number of feedback states does not change when the system has a zero in the forward path.

EXAMPLE 4.2 Consider the sixth order system in Example 3.7 with N-3 states available to be fedback. Recall that it was not possible to place dominant roots at s= $-2\pm j2$ with N-3 feedback states for this system using <u>only</u> transfer function methods. With a zero in the forward path of this system, the G(s)H(s) function is defined by equation 4.3.

$$G(s)H(s)= \frac{K(s+4)(s+2+j2)(s+2-j2)}{s(s+1)(s+5)^2(s+10)(s+50)} \qquad (4.3)$$

Equation 4.3 is equivalent to equation 3.11. As a result of the zero in the forward path of the system, a design problem with N-3 feedback states can be reduced to a design problem with N-2 feedback states.

C. CONCLUSIONS

The previous examples demonstrated that the designer's flexibility in satisfying system specifications is enhanced when the forward path of the system contains a zero. A zero in the forward path of the system defines a root location using design procedures with transfer function methods. The zero in the forward path of the system is a result of either the physical construction of the plant or by cascade compensation of the system. If the zero in the forward path of the system is due to the physical construction of the plant, it is assumed that the plant has been chosen to perform a particular task. If, however, the zero in the

forward path is a result of cascade compensation, the designer may choose the zero location such that it defines a desired root location. In both cases, the flexibility of the designer is enhanced using partial state feedback with transfer function methods when the system under study contains a zero in the forward path.

V. CONCLUSIONS AND RECOMMENDATIONS

State Feedback using transfer function methods is a very useful and effective design tool for automatic control systems. When all N states are available to be measured and fedback, the full state feedback design procedure using transfer function methods developed in Chapter II are equivalent to classical state variable root placement methods. Using the transfer function approach to control a system, only N-1 roots may be named by the designer, while the state variable analysis approach requires that all N roots be specified.

When fewer than N states are available to be measured and fedback, an observer is usually built in the feedback path to estimate the unaccessible states. The design procedures developed in Chapter III using transfer function methods does not require an observer. However, as the number of feedback states decreases, it becomes increasingly difficult to meet the required system specifications using the transfer function design technique presented in this paper. When the system specifications cannot be satisfied using the design procedure presented in this paper, the designer should consider a combination of compensation schemes to satisfy the system specifications. The results of chapter IV show that by adding a zero in the forward path of the system, the

designer's flexibility to satisfy system specifications are enhanced. The number of feedback states available is limited by those factors discussed in Chapter I. Additionally, the physical limitations of the plant will determine the amount of feedback that can be used in terms of system gain. These constraints will ultimately determine the type and amount of compensation available to the designer to control the system.

Both the full state and partial state feedback design procedures presented in this paper are effective tools in controlling an automatic control system. These techniques will not work for all systems, but should be considered by the designer with other compensation schemes in an attempt to control the system in the most efficient and cost effective manner. Two potential topics of further research for root placement using transfer function methods are:

1. To build and test a physical realization for a control system using the partial state feedback design techniques developed in this paper.
2. To develop design procedures for partial state feedback using a feedback filter. The zeros of the filter determine the system root locations, and the poles of the filter become the system's excess poles and are chosen significantly to the left of the jw axis.

APPENDIX A SECOND ORDER SYSTEM PARAMETERS

ZETA	$T_s W_n$	M_{pt}	M_{pw}	BW	PM_{rad}	PM_{deg}
0.04	100.00	1.88	12.51	1.55	0.08	4.58
0.06	66.67	1.83	8.35	1.55	0.12	6.87
0.08	50.00	1.78	6.27	1.55	0.16	9.15
0.10	40.00	1.73	5.03	1.54	0.20	11.42
0.12	33.33	1.68	4.20	1.54	0.24	13.68
0.14	28.57	1.64	3.61	1.53	0.28	15.94
0.16	25.00	1.60	3.17	1.53	0.32	18.17
0.18	22.22	1.56	2.82	1.52	0.36	20.40
0.20	20.00	1.53	2.55	1.51	0.39	22.60
0.22	18.18	1.49	2.33	1.50	0.43	24.79
0.24	16.57	1.46	2.15	1.49	0.47	26.95
0.26	15.38	1.43	1.99	1.48	0.51	29.09
0.28	14.29	1.40	1.86	1.47	0.54	31.19
0.30	13.33	1.37	1.75	1.45	0.58	33.27
0.32	12.50	1.35	1.65	1.44	0.62	35.32
0.34	11.76	1.32	1.56	1.42	0.65	37.33
0.36	11.11	1.30	1.49	4.41	0.69	39.30
0.38	10.53	1.28	1.42	1.39	0.72	41.23
0.40	10.00	1.25	1.36	1.37	0.75	43.12
0.42	9.52	1.23	1.31	1.36	0.78	44.96
0.44	9.09	1.21	1.27	1.34	0.82	46.75
0.46	8.70	1.20	1.22	1.32	0.85	48.50
0.48	8.33	1.18	1.19	1.29	0.88	50.19
0.50	8.00	1.16	1.15	1.27	0.90	51.83
0.52	7.69	1.15	1.13	1.25	0.93	53.41
0.54	7.41	1.13	1.10	1.22	0.96	54.94
0.56	7.14	1.12	1.08	1.20	0.98	56.41
0.58	6.90	1.11	1.06	1.17	1.01	57.83
0.60	6.57	1.09	1.04	1.15	1.03	59.19
0.62	6.45	1.08	1.03	1.12	1.06	60.49
0.64	6.25	1.07	1.02	1.09	1.08	61.74
0.66	6.06	1.06	1.01	1.07	1.10	62.93
0.68	5.88	1.05	1.00	1.04	1.12	64.07
0.70	5.71	1.05	1.00	1.01	1.14	65.16
0.72	5.56	1.04	1.00	0.98	1.16	66.19
0.74	5.41	1.03	1.00	0.95	1.17	67.18
0.76	5.26	1.03	1.01	0.93	1.19	68.12
0.78	5.13	1.02	1.02	0.90	1.20	69.01
0.80	5.00	1.02	1.04	0.87	1.22	69.86
0.82	4.88	1.01	1.07	0.84	1.23	70.67
0.84	4.76	1.01	1.10	0.82	1.25	71.43
0.86	4.65	1.01	1.14	0.79	1.26	72.16
0.88	4.55	1.00	1.20	0.77	1.27	72.86
0.90	4.44	1.00	1.27	0.75	1.28	73.51
0.92	4.35	1.00	1.39	0.72	1.29	74.14
0.94	4.26	1.00	1.56	0.70	1.30	74.73

APPENDIX B ASYMPTOTIC ANGLES FOR EXCESS POLES

EXCESS POLES	ASYMPTOTIC ANGLES
1	-180.0°
2	90.0° -90.0°
3	60.0° -60.0° -180.0°
4	45.0° -45.0° 135.0° -135.0°
5	36.0° -36.0° 108.0° -108.0° -180.0°
6	30.0° -30.0° 90.0° -90.0° 150.0° -150.0°
7	25.7° -25.7° 77.1° -77.1° 128.6° -128.6° -180.0°

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2. Friedland, Bernard, Control System Design; An Introduction To State Space Methods, pp. 243-246, McGraw-Hill Book Company, 1986.

3. Thaler, G. J., Automatic Control Systems, class notes for ECE-2300 class at the Naval Postgraduate School, Monterey, California, September 1987.

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